A Textbook of Logic

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Preface to the Fourth Edition

THIS fourth edition is yet another revised and enlarged edition of the earlier *A Text book of Logic — An Introduction*. Since incorrect use of language is one of the reasons for fallacious reasoning, it is necessary for a student to understand the use of right language in the formulation and evaluation of arguments. Therefore, a new chapter on "Uses and Functions of Language" has been included in the present edition.

I am thankful to my colleagues Dr Sneh Khosla, Dr Rajib Ray, Dr Raj Verma Sinha and Mansi Gupta for their help in many ways. I am equally thankful to Shri Susheel Kumar Mittal of the D.K. Printworld, Delhi for his keenness and personal interest shown in publishing this edition of the book.

Krishna Jain

Preface to the Third Edition

THIS is another revised edition of the earlier *A Textbook of Logic — An Introduction* (Revised and Enlarged Edition). Two new chapters, "Formal Proof of Validity" and "Predicate Calculus", have been added. Plenty of exercises have been given for the students to practise.

Krishna Jain

Preface to the Second Edition

THIS is an enlarged and revised edition of the earlier *A Textbook of Logic — An Introduction.* It contains, besides a new chapter on "Laws of Thought", many fresh exercises. I am confident that this new edition will serve the interest of the students better.

I am extremely grateful to my numerous friends, colleagues and students for their valuable and constructive suggestions and comments on the earlier edition of the text.

Krishna Jain

Preface to the First Edition

THE present book is the outcome of an interaction with students over a long period of time and proposes to explain the principles and procedures of Elementary Logic in the simplest possible way. It is an attempt to introduce students to both traditional as well as Symbolic Logic. It also covers Inductive Logic and includes Informal Fallacies committed in everyday arguments.

Almost all the topics are explained with the help of lucid examples. They also carry plenty of exercises for a better grasp of the subject. Special attempts have been made to clarify basic concepts such as Validity, Reasoning, Types of Reasoning, Proposition, Term, etc. In modern logic, Existential Import, Boolean Alzebra, Venn Diagrams, Truth Table, Shorter Truth Table Method are explained in a simple and easy language. A special chapter is provided for "translating" ordinary language sentences into symbolism of modern logic.

I shall like to express my gratitude to Prof. V.K. Bhardwaj, Deptt. of Philosophy, Delhi University, who went through the first draft of the text and offered many valuable suggestions. I am also thankful to Shri Balwant of Ajanta Books International for his keen interest in publishing the book. Last but not the least my thanks are due to my husband Dr V.K. Jain for encouraging me to write the present text.

Krishna Jain

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Chapter 1

Introduction

Subject Matter of Logic

Logic is a science of reasoning. The aim of logic is to provide methods, techniques and devices which help in differentiating right reasoning from wrong, and good reasoning from bad. But it does not mean that only those who study logic can reason correctly. However, it is true that those who study logic certainly make less errors while arguing. Just as a trained athlete is a better player than an untrained one, similarly a person acquainted with logical principles is likely to put forth good arguments. Knowledge of logic helps one to face a problem in a more orderly and systematic way, and in many cases makes the solution less difficult but more certain.

Science means a branch of coherently organized body of knowledge. Since logic is the study of consistent reasoning, it is certainly a science. Through logic we can judge, for example, whether a piece of reasoning such as we find in newspapers, magazines, etc. is correct or not, and also whether the conclusion follows correctly from the given evidences. Correct reasoning means to discover the right order between the evidences and conclusion. There is order and sequence in our reasoning. The moment one is concerned with the idea that one thought follows from another, he is being logical. Correct and consistent reasoning means conclusion follows from the evidences or the premisses. In other words, **correct reasoning means the premisses are strong enough to support the conclusion, and when the premisses are insufficient or inadequate to support the conclusion, then the reasoning becomes incorrect.**

Correct reasoning is the basis of all sciences, natural as well as social. In this sense it is very true to say that logic is presupposed by all sciences, and hence, it becomes a basic and primary science; a science of sciences.

But the logicians are not interested merely in the study of methods or techniques of differentiating right reasoning from wrong; it is equally important for them to acquire skill to apply these methods in determining the correctness of everyday reasoning and discourse as well. How efficiently or how skilfully one makes use of these methods in practical life is nothing but demonstrating the artistic aptitude. All arts are concerned with "doing" and "making". Anyone who knows logic "does" good reasoning and "makes" sound arguments. He makes good definitions and good debates. Logic prepares a man to make right reasoning and right decisions. "Practice makes a man prefect" is true for all arts, and it is equally true for logic as well.

Traditionally logic is defined as the study of laws of thought. There are three laws which have been considered indispensable for correct reasoning by the ancient logicians. They are :

1. Law of Identity
2. Law of Non-Contradiction
3. Law of Excluded-Middle

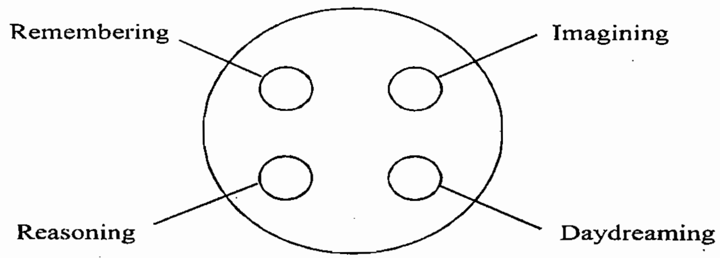
There is no doubt that these laws are absolutely indispensable for correct reasoning; but there are other laws also which are equally important for valid reasoning, for instance, various laws of inference and deduction.*

Moreover, in defining logic as the science of laws of thought, the word "thought" refers to thinking, to mental processes, and to mental activity. Whereas it is absolutely clear that reasoning is a reflective, a mental activity, and not a physical activity like walking, talking, etc.; yet every mental activity is not reasoning. There are many mental activities, and reasoning is one of them.

* These laws are discussed in detail in chapter 8.

In other words, reasoning is merely one of the mental processes. Every piece of reasoning is a mental process (thinking) but every mental process is not reasoning. Remembering, imagining, daydreaming are various types of mental processes (mental activities) but none of them is reasoning. The following diagram makes it clear:

Mental Processes



Reasoning is a special type of thinking, a special type of mental activity and a special type of mental process. When one thought is more or less consciously *connected* with another in order to elicit the conclusion towards which our thought is directed, then it is reasoning. For reasoning to take place there should be some basis, some ground, some evidences, some premisses which imply conclusion. In non-reasonical thinking no evidence is provided and no conclusion follows. Thus to define logic as the science of laws of thought is too wide a definition. Though it hints what logic is dealing with, yet it does not very specifically define it.

It is more appropriate to define **logic as the science of valid reasoning.** A logician, however, is not concerned with every aspect of reasoning. For instance, he is not dealing with reasoning as a "actual mental process". He is concerned only with its correctness or incorrectness. A logician looks to reasoning from a special angle, that is, from the viewpoint of its validity. A psychologist, on the other hand, studies reasoning from different angles. He studies all the mental processes, and in this sense,

studies reasoning also as one of the mental processes. But his interest is in the "actual mental process" while correctness of reasoning does not bother him at all. The logician's aim, however, is merely in correctness of arguments. His perspective is thus different from that of a psychologist. The latter is interested in knowing how actually do we reason and why; his study is thus only factual. The former, on the other hand, is interested in telling how we *should* reason correctly. For him norms, standards and criteria of correct reasoning are essential, since his interest is in improving man's habit of arguing.

Arguments

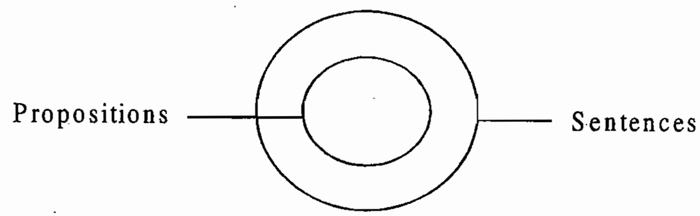
What does reasoning consist of? What is an argument? Reasoning means providing evidences for the conclusion in the clearest of term and supporting the conclusion with maximum evidences. The evidences provided to substantiate the conclusion are said to be premisses. That which is drawn on the basis of the premisses is a conclusion. An argument or a piece of reasoning is thus a relational arrangement of premisses and conclusion. An argument may have just one premiss or more but both premisses and conclusion should be there to form an argument. If the premisses are given and no conclusion is drawn from them, then it is not an argument. Similarly, if conclusion is given and no premisses are cited then also it is not an argument. Let us see some examples of arguments:

1. All men are mortal.
   Socrates is a man.
   Therefore, Socrates is mortal.
2. If there is rain, then we shall not go for picnic.
   There is rain.
   Therefore, we shall not go for picnic.
3. If A then B.
   If B then C.
   Therefore, if A then C.

4. All observed crows are black.
   Therefore, all crows are black.

It is very important that the student recognizes argumental form, and should be able to differentiate non-argumental sequence from a argumental presentation.

Both premiss and conclusion of an argument are propositions. A proposition is a sentence, but every sentence is not a proposition. Only informative or indicative sentences may be said to be propositions. Interrogative or exclamatory sentences are not propositions. A proposition is believed or disbelieved, asserted or denied whereas interrogative or exclamatory sentences are neither asserted nor denied. Questions are asked, commands are given and exclamations are uttered. But none of them can be affirmed, denied or judged to be either true or false. For instance, "Bring a glass of water" is not a proposition whereas "Water is essential for human beings" is a proposition. Hence, though every proposition is definitely a sentence, every sentence is not necessarily a proposition. Since logic deals with the proposition, so only indicative sentences find place in the preview of logic. This can be expressed as follows:



Besides, a proposition is language neutral entity. For instance, the sentence "This is a book" can be translated and expressed in various languages such as English, Hindi, German, Japanese, etc. But there is only one proposition underlying in all these differently expressed sentences. Logical propositions (statements) go beyond the language in which they are framed and expressed.

The unit of reasoning is a proposition which is totally and wholly language neutral. Both premisses and conclusion of an argument are propositions; but propositions by themselves do not carry any tag suggesting that certain propositions are exclusively premisses while some other propositions are exclusively conclusions. A proposition becomes either a premiss or a conclusion only in the context of an argument. Just as by entering into wedlock a woman becomes wife, similarly by entering into an argument some propositions become premisses and some other conclusions.

It is extremely important that we recognize which propositions are premisses and which one are conclusion in an argument. Without this any attempt to establish correctness of an argument is futile. There are words which help in identifying premisses and conclusions. The use of words like "hence", "therefore", "it follows", "thus", "so", etc. are conclusion indicators, and words or phrases like "because", "for", "since", "implies", etc. are premiss indicators.

Validity, correctness, rightness or soundness of an argument means premisses are sufficient and efficient enough to support the conclusion. A logician is confronted with such a question, "Are the premisses good evidences for accepting the conclusion?"If premisses adequately support the conclusion, then the argument is correct otherwise not. So evidences play a very vital role in determining the correctness of an argument. In scientific inquiries also the evidences play an extremely important role. In fact, all scientific explanations are essentially evidences based. Even in ordinary life often the evidences are produced to explain a certain situation, event or phenomenon. For instance, if a student comes late in the class, then on asking why he is late, he gives some explanation, some reason for his being late. Similarly, judges and detectives, historians and sociologists are looking and assessing the evidences for the events.

Form and Matter

Logic like pure mathematics is "science of forms" and norms. An argument consists both form and matter. The content of an argument is matter, and the order in which the content is arranged is form. Form, thus, is the manner in which all the constituents of the reasoning (premisses as well as conclusion) are arranged. The "form is not another constituent, but is the way the constituents are put together".[1]

Not only an argument but every object on the earth is made of some type of matter and it has some shape. Non-existent objects can also be conceived or thought in the *matrix of form and matter*. There is no matterless form and no formless matter. Both form and matter, however, are changeable. The same form can be given to different matter, for example same type of chair can be made from wood, iron, cane or plastic. Similarly different forms can be given to the same matter. For instance, to wood one may give the shape of chair, table, almirah, etc. Take the example of admission form of a college. All the admission forms are same, but the content in each of the form is different. The same is the case in currency. For instance, all ten rupee notes are similar in form but the number on each note is different.

Subject matter of an argument is the content. We may argue about political situations, educational policies, economic systems, etc. Form of an argument is order, pattern, arrangement in which all the elements (premisses and conclusion) stand in relation to one another. Form is like a model, a mold, a die in which different kinds of stuff can be put. Logical methods and principles which evaluate rightness of the arguments are constituted according to the forms of arguments. Innumerable arguments can be constituted having the same form. For instance, from the following argument, we can have many argument forms:

1. B. Russell, "Logic as the Essence of Philosophy", *Essay in Logic* (ed.) Ronald Jager, p. 126.

All M is P.
All S is M.
Therefore, all S is P

One can make innumerable arguments by giving different values to S, P, M. All those arguments will differ from one another in content only though they all share a common form; they all have the same arrangement of the premisses and conclusion. The "general pattern" of these arguments is the same. The form is "framework" of words like S, P, M occurring in the same position in different arguments.[2] Correctness or incorrectness of the argument is not decided by the values of words like S, P, M but by the manner in which they occupy the places in the premisses and conclusion.

The interest of all the deductive logicians lies in the form of an argument only because the correctness of an argument is to be decided solely by the form of the argument. The form of the argument must satisfy the logical principles in order to be correct (valid). Since the validity of an argument in deductive logic is totally decided by the form of the argument (content is absolutely irrelevant from the view point of validity), it is called a formal science.

The arguments in deductive or pure logic are classified into different categories according to the general form they possess. Arguments having same "pattern" and of the same "type" are treated similarly even if the content in each argument is different. The logicians are looking for "formal similarity" among the arguments. The logical principles or rules which determine correctness (validity)/ incorrectness (invalidity) of an argument is devised on the basis of "common logical features" of the argument. Thus, the logical form of an argument is the most precious thing in logic. The formal logicians frame rules and formulae for testing the validity of arguments on the basis of logical form they possess. They (the logicians) are thus hunting

2. Cf. P.F. Strawson, *Introduction to Logical Theory*, p. 45.

for "formal analogy" among the arguments. The arguments having same "framework" of occurring words and terms are tested by the same logical methods.[3] Logicians' task consists in "compiling list of highly general rules" of reasoning and arguments. They (the logicians) provide "representative formulae" for testing the validity/invalidity of the arguments. The arguments having the same kind, same type will be examined and tested by the same general formulae. This makes the logicians' job a little simpler. For example, if there is rule for each concrete individual argument, then "the principles of logic will be longer than a dictionary". In order to avoid that, all arguments of the same type are bracketed together and are thus handled together. The "representative formulae" or the "general rules" of arguments are, however, not inconsistent to each other. They are connected and provide ideal and organic system of logic.

Truth and Validity

Two sets of evaluative notions are generally used in elementary logic: true/false and valid/invalid. A proposition is either true or false. A proposition is true when it corresponds to reality, "when it mirrors the world". If, on the other hand, a proposition states the facts incorrectly, then it is false. For example, "All men are mortal", "All cats are mammals", "All cows are four legged animals" are true propositions for they describe the facts correctly whereas propositions like "All fruits are sweet", "All Indian women are literate" are false propositions for they describe the facts wrongly. A proposition is however, never evaluated as valid or invalid.

A deductive argument instead is evaluated as valid or invalid and never as true or false. An argument is valid if its premisses necessarily imply the conclusion. In other words, when the conclusion necessarily follows from the premisses, then the argument is considered valid. Validity and invalidity are formal notions and hence are applied to formal reasoning and formal logic only.

3. Ibid., pp. 45-46.

Though we can isolate these evaluative notions (truth and falsity, valid and invalid) from each other, yet they are not unconnected. There is a relation between truth and falsity of the premisses and conclusion, on the one hand, and validity or invalidity of an argument, on the other hand. In fact, the entire study of logic is intended to explore the relationship between truth and falsity of propositions, and validity and invalidity of arguments. **Generally, it is believed that a valid argument has a true conclusion and invalid argument has false conclusion. But it is wrong.** Thinking of such kind is source of serious errors in logic. The true conclusion does not necessarily mean the argument is valid. Similarly, the false conclusion does not necessarily mean the argument is invalid. A valid argument can have a false conclusion or invalid argument can have true conclusion. But valid arguments having a false conclusion must have at least one false premiss. **If all the premisses are true and the conclusion is false, then the argument is definitely invalid. In fact, the chief characteristic of deductive logic is that it is impossible to have all true premisses and a false conclusion.** A deductive argument takes the form. "If you accept these premisses as true, then you must accept this conclusion as true as well".

Let us now examine the various situations in which the truth and falsity of premisses and conclusion make the argument valid or invalid.

Consider a situation in which all propositions, premisses as well as conclusion, are true. In certain cases they yield a valid argument and in some other an invalid argument depending on the arrangement of premisses and conclusion. For example :

All human beings are mortal.
All professors are human beings.

Therefore, all professors are mortal,

is a valid argument having all true premisses and a true conclusion. But the argument.

All cats are mammals.
All cats are rat-eaters.

Therefore, some rat-eaters are mammals,

is invalid in spite of the fact that all premisses and conclusion are true. Take another example:

If today is Sunday, then tomorrow is Monday.
Tomorrow is Monday.

Therefore, today is Sunday,

is invalid in spite of the fact that all premisses and conclusion are true. Consider yet another example:

If I am 18 years old or above, then I am a voter in Indian general election.
I am voter in Indian general election.

Therefore, I am 18 years' old or above,

is invalid argument in spite of the fact that all premisses and conclusion are true.

On the other hand, all false propositions (premisses as well as conclusions) may make an argument valid. For example :

All vegetarians can fly.
All cats are vegetarians.

Therefore, all cats can fly,

is valid argument in spite of the fact the premisses and the conclusion are all false.

But the argument:

All cats are wild creatures.
All horses are wild creatures.

Therefore, all horses are cats,

is invalid having all false propositions.[4]

The combination of false premisses and true conclusion may make an argument valid or invalid depending on the arrangement of premisses and conclusion. For example :

4. Why and how the argument is invalid is discussed in chapter 7.

All bricks are combustible.
No paper is combustible.
Therefore, no paper is brick,

is valid argument having false premisses but a true conclusion. Similarly,

No living creatures need air to breath.
All stones are living creatures.
Therefore, no stone needs air to breath,

is valid though both the premisses are false, and the conclusion is true. But,

All literate beings are women.
All Indian Prime Ministers are women.
Therefore, all Indian Prime Ministers are literates,

is invalid having all false premisses and a true conclusion.

If all the premisses are true and the conclusion is false, then a deductive argument is bound to be invalid. In fact, the chief characteristic of deductive reasoning is that it is impossible to have all true premisses and a false conclusion. A true proposition or a set of all true propositions can imply only true propositions.[5] For example in the argument:

All cows are four-legged creatures.
All horses are four-legged creatures.
Therefore, all horses are cows,

both the premisses are true but the conclusion is false hence the argument is invalid. Why all true premisses and false conclusions make the argument invalid? There is a reason. In the formal logic the conclusion is already contained in the premisses. The premisses inherit the conclusion in the implicit form. The conclusion does not go beyond the premisses. Thus, if the premisses are true, the conclusion must necessarily be true, for the conclusion says nothing new or different from the premisses.

5. W.V. Quine, *Methods of Logic,* p. 4.

To have all true premisses and false conclusion means to contradict one self. We accept the truth of certain facts in the premisses and in the conclusion we deny the same thing. How can this be possible? It is inconsistent to assert and deny the same thing. No one, specially the logician, must not contradict himself. Thus, "an argument is valid only if it would be inconsistent (or self-contradictory) to assert the premisses while denying the conclusion".[6]

From the above discussion it is absolutely clear that the relationship between truth and falsity of propositions on the one hand, and validity and invalidity of the argument on the other hand, is of "peculiar type". In the following chapters you will learn this relationship in detail.

Deduction and Induction

Two types of relationships that are recognized by the logicians between premisses and conclusion are deductive and inductive. Consequently, there are two types of reasoning and two types of logic: deductive and inductive. Though the nature of reasoning in both of them is different, their aim is nonetheless same. **Both deductive and inductive logic provide methods and criteria to differentiate correct reasoning from incorrect ones.**

The relationship between the premisses and the conclusion in deductive reasoning is of implication and "entailment". The implicative and "entailment" relationship justifies the assertion that the conclusion necessarily follows from the premisses in accordance with the logical principles. The conclusion in deduction follows necessarily from the premisses because the conclusion is already inherited in the premisses. For instance in the argument.

All men are mortal.
Socrates is a man.
Therefore, Socrates is mortal.

6. P.F. Strawson, op.cit. p. 2.

The conclusion "Socrates is mortal" is already inherited in the premisses "All men are mortal" and "Socrates is a man". The conclusion is present in the premisses in the implicit form. The reasoning makes the implicit form of the conclusion explicit. The conclusion, thus, provides no new information.

Besides, the set of premisses "All men are mortal" and "Socrates is a man" are *conclusive* ground for the conclusion "Socrates is mortal". Conclusive ground means complete evidence, sufficient and total evidences to support the conclusion. Only deductive logic has the privilege of supporting the conclusion with conclusive evidences. In the above-cited example, the conclusion "Socrates is mortal" follows necessarily from the premisses "All men are mortal" and "Socrates is a man". Any addition to already given set of premisses, for instance "Socrates was a philosopher", "He was a Greek", "He was a teacher of famous philosopher Plato", etc. makes no difference to the conclusion "Socrates is mortal". The given two premisses "All men are mortal" and "Socrates is a man" are sufficient and enough to imply the conclusion "Socrates is mortal". In this sense and only in this sense evidences in deductive reasoning are conclusive and complete.

As we have seen the conclusion is already inherited in the premisses, it is impossible then for a valid deductive argument to have all true premisses and a false conclusion. All true premisses implying false conclusion means to contradict oneself. Since the conclusion is already present in the premisses and all premisses are true, then how can the conclusion be false without "contradictory oneself".

But very often the conclusion of an argument does not stand to the premisses in so necessary a logical relation as that of being implied or entailed by the premisses. Most of the reasoning that people do in everyday life is non-deductive (inductive). Doctors use non-deductive reasoning in diagnosing the probable causes of a patient's symptoms. Legal scholars often use non-deductive methods to determine what law governs a particular case. Non-

deductive or inductive arguments are tentative, provisional and probable.

There is nothing contradictory in accepting all true premisses and a false conclusion in non-deductive arguments. For instance in the following example.

Professor X is a writer and he is rich.
Professor Y is a writer and he is rich.
Professor Z is a writer and he is rich.
Therefore, all professors who are writers are rich,

All the premisses are true but the conclusion is false, and still it is not contradictory to accept the truth of the premisses and falsity of the conclusion. Thus the chief characteristic of deductive logic that it is impossible to have all true premisses and false conclusion is **not** applicable to inductive reasoning.

Moreover, in the inductive inferences the evidences do not conclusively support the conclusion. The evidences may be very weighty, very strong still they are inconclusive ground for the conclusion. They (evidences) do not imply the conclusion with utmost certainty. That is why, the conclusions in inductive reasoning are merely probable. Look at this example:

Crow X is black.
Crow Y is black.
Crow Z is black.
Therefore, probably all crows are black.

Here one can never be sure that all crows are black. It is possible that in future one may come across a crow which is of non-black colour. Since one cannot conceptually rule out the possibility of non-black coloured crow, the conclusion "All crows are black" is merely probable. This means the conclusion and the premisses are not related by strict implicative relation in inductive logic. The evidences may be very strong in favour of a certain conclusion still the conclusion is not as certain as we find in deductive reasoning. The conclusions of experimental sciences have very high degree of probability but they still are not certain

in the strick sense of the term. The strength of the conclusions of inductive reasoning vary in degrees: some conclusions are highly probable whereas some others are less probable. Evidences also vary in their strength. Evidences in some cases are very strong, "overwhelming" while in other cases they may be merely "slender" or "poor". The evidences (premisses) only support the conclusion and do not "entail it". That is why the evidences in inductive inferences (reasoning) are inconclusive and incomplete.

An inductive inference is neither true nor false; also it is neither valid nor invalid either. An inductive argument (inference) is characterized or evaluated as sound or unsound, good or bad, right or wrong, appealing or non-appealing, convincing or unconvincing, etc. The relative notions which carry degrees of changeability as good/better, convincing/not so convincing, appealing/not so appealing characterize inductive inferences. Inductive arguments do not differ among themselves in kinds; they differ only in degrees. No inductive argument is totally valid or invalid: it can only be better or worse. But in deductive arguments there is a clear polarization. They differ in kinds, and not in degrees. A deductive argument is either valid or invalid. The phrase "moderately valid" is not for deductive arguments whereas it can be applied to inductive reasoning very comfortably.

The basic principle behind every deductive argument is that conclusion can never be wider than the sum total of the premisses. In a valid deductive argument a conclusion is either equal or less than the premisses whereas the conclusion in a sound inductive argument can be more, less or equal to the premisses.

Deductive logic like mathematics is a formal science. The "form" of an argument alone decides the validity of an argument; the matter has absolutely no role in determining the validity of an argument. The "pure form" of an argument is ideal for the deductive logicians and they are interested only in that because validity of an argument depends on it.

Thus, while form is the only deciding factor in assessing the validity of a deductive argument, it is both form and matter which evaluate inductive inferences. The logical form of an inductive inference is as follows:

1. All observed crows are black.
   Therefore, all crows are black.
2. All observed Ms are Ns.
   Therefore, all Ms are Ns.

The above argumental form, however, may not be true for all arbitrary values of M and N unlike the case of deductive argument.

Moreover, a deductive logician does not question the status of the premisses; he does not bother whether the given premisses are actually true or false. The premisses are "given" to him and he takes for granted their truth value and accept them as they are. The job of a deductive logician is merely to examine what necessarily follows from the "given" set of premisses. In the case of inductive reasoning, however, the job of a logician is not that simple. The premisses are not "given" to him. The inductive logician hunts and collects the premisses which are in the form of evidences and data. The evidences are actually experienced events which are witnessed by the arguer or by some other being. After collecting sufficient number of data or evidences, the arguer establishes conclusion. For instance, from the evidences that all the crows observed so far are black, one establishes that "All crows are black". But the conclusion is merely **established** and **not entailed.** Entailment relation is a strict and necessary relation which is not applicable to any of the inductive inferences.

In other words, it is "reasonable" and "not rational" to establish a conclusion on the basis of collected data. It is only reasonable to draw the conclusion "All crows are black" on the basis of "All *observed* crows are black". In logic "being rational" means being

"formally valid" whereas being reasonable means acceptably sound. Non-deductive inferences (inductive inferences) can at best be reasonably sound and not rationally valid.

Thus, the job of a inductive logician is twofold: first to collect the *data* (premisses) and second, to establish a reasonably acceptable conclusion on the basis of collected data. But the deductive logician has only one task, that of examining what follows necessarily from a given set of premisses.

It is, however, very wrong to consider induction and deduction as opposed reasonings. They are not to be considered as contradictory to each other but rather as complementary and supplementary to one another. The conclusion of inductive reasoning may serve as the premiss in deduction. The relation between deductive and inductive reasoning is more like a relay race. Deduction begins where induction terminates. They differ only in their "starting points".

Just as deductive logic has affinity with mathematics, inductive logic has with the methods employed by the scientists. A question may arise; why a logician is interested in the scientific methods? A logician is interested in the scientific methods and scientific methodology because a scientist like a logician is concerned with the evidences which support the conclusion.

Chapter 2

Functions and Uses of Language

LANGUAGE is an important vehicle through which ideas and concepts are transmitted. It is a primary source, a common platform and an indispensable medium of social interaction. The study of language is an inquiry into the human minds and language are considered "the best mirrors of the human minds". Language has a profound influence on human development and use and misuse of language has been a fundamental factor in shaping human history. A question may arise: why is a logician interested in the study of language? The answer is simple. Language is both "aid and obstacle in reasoning". Logic is study of arguments, and arguments are expressed through language. Logician's chief aim is to determine the validity/invalidity of arguments. The validity of arguments depends on the correct form of the arguments. The correct form of arguments, in turn, requires clear and precise use of language. Incorrect use of language, on the other hand, leads to fallacious reasoning. Thus, the study of language is all the more important to a logician because the purpose of logic is to improve our critical thinking. To think critically is to recognize, analyse, construct and evaluate arguments.

Language Makes Thinking Possible

It is an established fact that there is a close relation between language and thinking. "Language makes thought possible. Learning a language is not just learning a new way to put our thoughts into words; it is also learning a new way to think."[1] Without words or symbols we cannot think. Our thinking becomes

1. Gilbert Harman, *Thought*, pp. 84-85.

intelligable only in the matrix of language. Even in Soliloquy where one talks to ones ownself, the use of language is indispensable. An expressive summary of this is found in Shelley's remark. "He gave men speech, and speech created thought . . .".[2] Language and thought are interdependent. But the thinking in which the logicians are interested in is "reasoned thinking which takes place as we work out problems, tell stories, plan strategies, and so on. It has been called "rational', "direct', "logical" or "prepositional" thinking. It involves elements that are both deductive (when we solves problems by using a given set of rules as in arithmetical task) and inductive (when one solves problems on the basis of data placed before us, as in working out a travel route). Language seems to be very important for this kind of thinking. The formal properties of language, such as word order and sentence sequencing, constitute the medium in which our connected thought can be presented and organized.[3]

It is very important for a student of logic to learn what conclusion follows from a given set of propositions (premisses). He should know how to organize his thoughts in the sequence of given facts (the basis, the premisses) and the inferred facts (conclusion).[4]

Various Functions of Language

Language serves numerous functions for human beings. We use language in our everyday life to communicate information, describe objects, to greet people, to let others know our requirements, to express our feelings, to request, to give instructions, to sing, to crack jokes, and many more things. There are "countless" uses of language. Ludwig Wittgenstein, a famous twentieth-century linguistic philosopher, has described language

2. Cf. David Crystal, *The Cambridge Encyclopedia of Language* (second edition) p. 14.
3. Ibid.
4. Cf. Noam Chomsky, *Knowledge of Language (Its Nature, Origin and Use)*, p. xxvi.

as a "tool", as an "instrument". Using language is like playing games, "language-games".[5] Language-games are not part of any theory about language, but are one way of looking at language as a phenomenon.

While studying language it is important to analyse the tasks performed by language and kinds of sentences used in performing these tasks. In ordinary discourse, language serves multiple functions, and in determining its specific use one must consider its context and purpose of the speaker. The grammarians divide the uses of language into such classes as directive, expressive, evaluative, cognitive, ceremonial, and so on. But three functions of language are specially worthy of note. Identifying these three functions is simple but their awareness is important to understand complexity of language. These three functions are:

(i) Informative function
(ii) Expressive function
(iii) Directive function

INFORMATIVE FUNCTION

The first basic function of language is to inform, to report and to describe facts. We communicate our beliefs and opinions with others which may be true or false. See the following examples:

"Socrates was a Greek philosopher".
"New Delhi is Capital of India".
"Shakespeare was the author of *King Lear*".
"Mercury rises with heat".
"Two plus two is four".

All the above statements are informative and purpose of these statements is to state "what the facts are." We may believe or disbelieve, assert or deny them. The information provided may be correct or incorrect. The main characteristics of these statements

5. Ludwig Wittgenstein, *Philosophical Investigations*, tr. G.E.M. Anscombe, Oxford: Basil Blackwell; New York: Macmillan, 1958.

are that they declare whether facts exist or not. They deal with dichotomical statements true or false.

Logicians and scientists are concerned exclusively with informative use of language. A scientific statement is exact and precise and it carries truth-values. There are two truth-values true and false. However, a scientific statement cannot be both true and false at the same time; also it cannot be neither true nor false at the same time. Exactitude is the characteristic of the scientific statements and only informative use of language makes statements with exactitude. Like scientist a logician is also concerned with precise use of language in affirming or denying propositions, formulating arguments, evaluating them and so on. The criterion of truth and falsity is attached to them, and only informative use of language can perform this task. Logicians and scientists thus are interested exclusively in this function of language.

EXPRESSIVE FUNCTION

We have seen language has an indefinite number of uses in addition to the communication of information. Many of these uses are non-logical. Not always we use language to evaluate the facts as true or false. Sometimes the sentences are made not for the sake of communicating information, but in order to arouse in the listener a certain response, to make them do some work, or sometimes even to perform some act. Those statements are certainly neither true nor false.

The expressive use of language employs words, phrases, or even sentences to show the feelings, emotions, attitudes, and dispositions of the speaker. Through language we express our joys and sorrows, pleasure and pain. A player in a match expresses his pleasure when he utters "We have won"! See the following examples:

"Rainbow is beautiful"!
"Cricket is a great play"!
"The portraits are beautiful"!

"Oh! What a relief"!
"Fantastic"!
"Oh! That's great"!
"Terrific"!
"Oh! My God"!
"You Bravo"!

These statements express feelings and emotions. Expressive language can be used effectively to arouse feelings of love, anger, hate, etc. in the minds of listeners. Lyrical poetry is very good example of expressive function of language. Poems are not meant to report any information but to express the feelings that the poet feels keenly about. No criteria of truth or falsity are applied to emotions or feelings because they are neither true nor false. The criteria of truth and falsity is not applied to directive function of language either because commands and orders are meant to be obeyed and no one ask about their truth or falsity.

DIRECTIVE FUNCTION

Sometimes language is used to get a particular work done. Sentences and words cause overt actions. By means of language one can command, direct, request or suggest others to do some work. One can compel others to perform or to prevent an action. Command forms large group of directive expressions. To ask questions is also directive use of language because the function of a question is to ask for an answer. See the following examples:

"Close the door".
"Please give me your pen".
"I suggest you to take up this study course material".
"Is tomorrow a holiday"?
"Right turn"!
"Please pass this tray".
"When do the classes begin"?
"Attention".
"Brush your teeth twice a day".
"Come here"!

No one asks whether these statements are true or false because the criterion of truth/falsity is not applied to them.

Besides communicating, reporting, describing, commanding, language serves other functions also, for instance ceremonial and performative functions. Though it is true that ceremonial discourse can "always be regarded as a mixture of expressive and directive discourse, rather than as a separate kind that is unique"[6] yet it needs to be mentioned here separately. To greet a friend using words or phrases like "Hey", "Hello", "Good morning", "How are you", etc. are some of the illustrations of the ceremonial discourses. Here we refer to phrases or sentences "ranging from trivial words of greeting to the verbal rituals performed on holy days in houses of worship".[7] Ceremonial phrases are important for "goodwill and sociability". Sentences of this kind are, usually, automatically produced and are of stereotyped structure. The most common linguistic expression of emotion consists of conventional words or phrases such as "Gosh", "What a sight"! The ritual exchanges about health or the weather is not to communicate ideas but to maintain "comfortable relationship between people".[8] Ceremonial function thus can hardly be informative for those who use it.

A performative expression, on the other hand, is the one that is used to accomplish some social acts, in contrast to reporting, evaluating, reacting to it, etc. For instance, while performing marriages the priest's discourses are performative in nature. Casting a vote is another example of performative function. Yet another example of this category is party saying "I accept the offer" or "I accept the proposal".

Conclusion

By this time you must have noticed that language is very flexible and it may serve many functions simultaneously. For example,

6. I.M. Copi, *Introduction to Logic* (ninth edition) p. 84.
7. Ibid.
8. David Crystal, op. cit., p. 10.

a traveller tells his friend "The train leaves in ten minutes". Such a statement is not only informative but is designed to influence the action of the friend as well. Any ordinary communication exemplifies all the three basic functions of language to a greater or lesser extent. This "mixed use" of language recognizes that a given expression may have multiple uses in a given context. For example: "This building is on fire"! indicates many functions at the same time.

The three basic functions of language (informative, expressive and directive) thus are not mutually exclusive. They are overlapping functions and a statement which is used informatively, can also be used expressively, or what can be used directively, can also be used expressively; and so on. The descriptive purpose in language may come in conjunctions with other purposes of discourse as well. However, the logician's chief concern would be to identify the informative discourse only. A student of logic should be able to separate it from its expressive, directive, ceremonial or performative uses.

Arguments sometimes became fallacious because the meanings of the terms and sentences are not properly known. Fallacies such as those of *Relevance* (which provide a weak link between premisses and conclusion) or of *Ambiguity* (which occur because of the use of expressive language) make arguments invalid. Since logic is concerned with what is inferred and on what ground it is inferred, the logician aspires only for emotionally neutral language.

Chapter 3 – Section A

Proposition
Traditional Account

A PROPOSITION is unit of reasoning in logic. Both premisses and conclusion of an argument are propositions. A proposition is a sentence, a meaningful arrangement of words. It has a subject, a predicate and a copula. Every sentence, however, is not always a proposition. Informative, indicative or factual sentences alone are propositions. Interrogative, exclamatory, emotive, or imperative sentences are not.

A proposition is a sentence which is either true or false, asserted or not asserted, believed or disbelieved, supposed or doubted. Whereas factual or indicative sentences are true or false, the imperative, exclamatory or interrogative sentences are neither of them. For example, when one utters "What a beautiful rainbow!" no one expects it to be true or false. The uttered sentence is merely an expression of a feeling. Similarly, if the father orders the child to close the TV, the uttered sentences does not carry any true value. It is an order to be obeyed. Questions may be asked, commands given, and exclamations uttered, but none of them can be affirmed, denied or judged to be either true or false.

A proposition is true when it describes the facts correctly; and a proposition is false when it does not describe the facts correctly. One may believe a sentence to be either true or false. In this capacity the believed or disbelieved sentences become propositions for they carry truth values. Mythical statements too are propositions because they are believed to be either true or false. "Ram has killed Rawan" is a proposition for it is believed to be true. "Padmani was a beautiful lady" is a proposition for it is

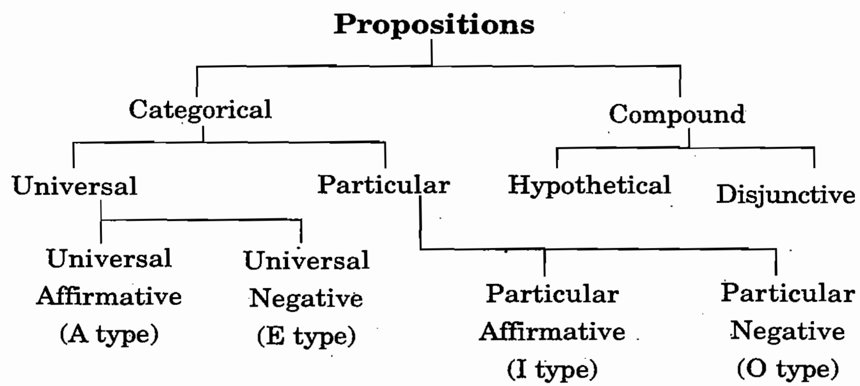
believed to be true, but "P is beautiful" is not a proposition because it is neither true nor false.

A proposition which is always true is called *tautology* in logic. For example, "The sum of all angles of a triangle is equivalent to 180°", "Cats are mammals", "No man is immortal", etc. are tautologous propositions. A proposition which is always false is called *contradictory* or *self-contradictory*. For instance, "All men are immortal", "All circles are polygon" are self-contradictory propositions. However, when a proposition is true in some cases and false in some other cases, it is called *contingent*. For instance, the proposition "It is raining" is true just now but it may be false some other time.

Traditional Classification of Propositions

Traditional logicians[1] have divided the propositions into two groups, categorical and compound. A proposition is either categorical or compound. The modern logicians,[2] however,

Traditional classification of propositions



1. The ancient Greek philosopher Aristotle was the founder of logic as the science. He was one of the first scholars to carry out a systematic study of the propositions and arguments.
2. Modern logic extends far beyond the work of traditional Aristotelian logic. Modern logicians unlike the traditional logicians have used mathematical methods and techniques to test the validity of arguments. Some of the leading modern logicians are British mathematicians such →

classified them differently. Let us first examine the traditional classification of propositions.

Categorical Propositions

Categorical propositions are stated without any condition. Here the predicate is either assigned or denied to the subject without any condition. For instance, "All men are mortal", "This chair is round", "Some students are not scholarship holders" are categorical propositions. A conditional proposition, on the other hand, is that in which a predicate is assigned or denied to subject on certain conditions. For instance, "If we get tickets, we shall go for movie", "Sita will not get job unless she hunts for it", "If Ram is intelligent then he will qualify the test", are conditional propositions.

Aristotle and other classical logicians divided the categorical propositions into four types:

1. Universal affirmative — All S is P — All men are mortal. "A" type proposition
2. Universal negative — No S is P — No fish is mammal. "E" type proposition
3. Particular affirmative — Some S is P — Some men are wise. "I" type proposition
4. Particular negative — Some S is not P — Some students are not voters. "O" type proposition

"S" is subject and "P" is predicate of the proposition

"A" type proposition is universal affirmative.
"E" type proposition is universal negative.
"I" type proposition is particular affirmative.
"O" type proposition is particular negative.

All these propositions A, E, I, O have only one form that is "subject predicate form". The traditional logicians have recognized

→ as George Boole, Alfred North Whitehead and British Philosopher Bertrand Russell. Today, logic is used mainly to test the validity of all kinds of arguments. It also has important uses in working with such devices as computers and electric switching circuits.

only one form for the categorical propositions. There are three constituents of a categorical proposition — subject, predicate and copula. In a proposition "All S is P", "S" is subject, "P" is predicate and "is" is copula. Both "S" and "P" are nouns and they represent classes.

QUANTITY AND QUALITY

The traditional logicians have introduced two more notions for the categorical propositions. They are quantity and quality.

Quantity of a proposition is related to the generalization of the subject of a proposition. The generalization can be unrestricted or restricted. Hence, there are two quantities — universal and particular. Subject of a universal proposition indicates unrestricted generalization, and in a particular proposition subject indicates restricted generalization. For instance, in the proposition, "All crows are black", the subject term "crows" denotes the entire class of crows. In the proposition, "All men are mortal", since the entire class of "man" is taken into consideration, the quantity of the proposition is universal. In the proposition "No fish is mammal", again the quantity is universal. Here the entire class of fish is excluded from the entire class of mammals. However, when the subject of a proposition indicates only part of a class, the quantity of the proposition is particular. For instance, in the proposition "Some students are scholarship holders", the quantity is particular and similarly in the proposition, "Some men are not happy beings", the quantity is particular.

Quality of a proposition means whether the predicate is assigned or denied to the subject. There are two qualities which a proposition can have; affirmative or negative. If a predicate is assigned to subject then quality of the proposition is affirmative. For instance, in the proposition, "All flowers are beautiful", the predicate "beautiful" is assigned to the subject "flowers". If a predicate is denied to the subject, then quality of the proposition is negative. For instance, in the proposition "Some men are not

wise", predicate "wise" is denied to the subject "men". However, a negative term like "immortal", "unwise" etc. do not make a proposition negative. It is negative copula which makes a proposition negative. For example, "Some men are unwise", is an affirmative proposition with a negative predicate whereas "Some men are not wise", is a negative proposition.

Quantity and quality together give four types of propositions:

1. Universal affirmative
2. Universal negative
3. Particular affirmative
4. Particular negative

In "A" proposition, "All men are mortal", quantity is universal and quality is affirmative. In "E" proposition, "No fish is mammal", quantity is universal and quality is negative. In "I" proposition, "Some students are hardworking", quantity is particular and quality is affirmative. And in "O" proposition, "Some men are not wise", quantity is particular and quality is negative.

A singular proposition, in the traditional classification of proposition, is included under universal proposition. In singular proposition subject is either a proper noun or refers to specific person or object. For example, "Socrates was a Greek philosopher", "This is a girl's college", "New Delhi is capital of India", are few instances of singular propositions. No independent status is given to such propositions by the traditional logicians; these propositions are bracketed with universal propositions. "Socrates was a Greek philosopher", is universal affirmative proposition whereas "Socrates was not an Indian", is universal negative proposition. This was, however, challenged and criticized by the modern logicians which you will study later in this chapter.

Why were the singular propositions treated as universal propositions by the classical thinkers? It is because in spite of the fact that the subject term of the singular proposition is a "specific individual or object", it represents a class, a unit class[3] (one membered class)[4]. There is only one member in the class referred by the subject Socrates in the proposition "Socrates was a Greek philosopher". Since the predicate in this proposition is assigned to that very one member of the class, it becomes "A" proposition. "Socrates was not an Indian", is "E" proposition for the similar reason. The predicate "an Indian", is denied to the subject "Socrates", which incidentally is only one membered class.

Traditionally all the categorical propositions are divided into four groups whose standard logical form is as follows :

All S is P	-	A
No S is P	-	E
Some S is P	-	I
Some S is not P	-	O

All the simple, singular, unconditional categorical propositions are one of the above four types.

Reduction of the Sentences into Standard Logical Form Propositions

In our ordinary life and also in various branches of sciences, sentences are not always in this neat and formal logical form. For instance sentences like:

"All roses are not red".

"An elephant is a vegetarian animal".

"Many students are not interested in the formal education".

are not in the standard logical form. They are to be translated into the standard logical form propositions. The above propositions are translated as :

3. "Woman Prime Minister of India up to year 1982" is an example of unit class. Since there is one member in the class of Woman Prime Minister of India, it is called unit class.
4. Cf. Irving M. Copi and Cohen Carl, *Introduction to Logic* (Ninth Edition), p. 278.

"Some roses are not red things".

"All elephants are vegetarian animals".

"Some students are not interested in the formal education".

respectively. The translation or the reduction of non-standard form propositions into standard form propositions is important because all the methods and techniques to determine the validity of arguments are in terms of standard form propositions. There are various devices to reduce a non-standard form proposition into standard form proposition. However, the meaning of the proposition should not be lost in this process. This is very important. In fact, the meaning of the proposition helps in the reduction and translation. Some of the devices and techniques are as follows:

Device 1. Both subject and predicate of a standard form categorical proposition are substantive nouns. If either of them (subject or predicate) is not noun then it has to be translated into appropriate noun.

The proposition "Some roses are beautiful", is translated as "Some roses are beautiful things".

"All cats eat meat", becomes "All cats are meat eaters".

"All chairs are meant for sitting", is reduced as "All chairs are things meant for sitting".

"Ram plays football", is translated as "Ram is football player".

"Violence breeds violence", becomes "All acts of violence are violence breeders".

The above examples show that if predicate is a verb or an adjective, then it should be replaced by substantive noun. In a standard form categorical proposition there should be one subject term (noun) and one predicate term (noun), and a copula to join them. Both subject and predicate should necessarily be substantive noun. It is because both subject and predicate represent classes and only substantive nouns can do that.

Device 2. Propositions like :

"All that glitters is not gold".

"Not all roses are red".

"Every student is not scholarship holder".

"Not every inexplicable event is miraculous event".

are not universal negative propositions. They deceptively suggest that they are universal negative, but their meaning indicates that they are only particular negative propositions. After translation the above propositions become:

"Something that glitters is not gold".

"Some roses are not red things".

"Some students are not scholarship holders".

"Some inexplicable events are not miraculous events".

respectively.

Words like "hardly", "rarely", "seldom", "scarcely", "few" and "little", "Not always", "not every where", "some times not" are indicators of particular negative propositions. But the words like "never", "no where", "under no circumstance" indicate universal negative propositions.

Device 3. There are propositions having words like, "only", "none but", etc. For example :

"Only policemen are indispensable".

"None but scholarship holders are invited for the discussion".

can be translated into universal affirmative "A" propositions provided subject term and predicate term interchange their places. The above propositions are reduced as follows :

"All those who are indispensable are policemen".

"All those who are invited for the discussion are scholarship holders".

Though this type of reduction is quite popular but there are other ways also in which "only", "none but" propositions can be translated. The proposition, "Only policemen are indispensable", can also be translated into "E" proposition as, "No non-policemen

are indispensable", and "None but scholarship holders are invited for the discussion", can be reduced as "No non-scholarship holders are invited for the discussion".

But proposition like "Only some students are hard working", is translated differently from what we have done above. Such translations you will find in device number 9.

Device 4. There are propositions having no quantifiers at all. For instance:

"Children are naughty".
"Students are present".
"There are honest politicians".
"Flowers are beautiful".

The meaning of these propositions reveal their quantity. The above propositions are translated as follows:

"All children are naughty".
"Some students are persons who are present here".
"Some politicians are honest beings".
"All flowers are beautiful things".

Device 5. There are words which indicate universal quantity such as "all", "any", "every", "always", "everywhere", "in every instances", "who", "whatsoever", etc. For instance the propositions:

"Any person is free to be a member of this Library".
"Every man loves children".
"Women always are delicate".
"Each application was checked".

are translated as :

"All persons are eligible to be the member of this Library".
"All men are children lovers".
"All women are delicate beings".
"All applications are things checked". respectively.

Device 6. But the quantity of the propositions having words like, "a" or "an" is determined by the context in which they occur. For instance, proposition like. "A snake is reptile", has different

quantity from the proposition "A snake was killed by the gardener yesterday". Whereas, the former suggests universal affirmative proposition, "All snakes are reptile creatures", the latter is a particular affirmative proposition, "Some snake was a creature killed by the gardener yesterday". Similarly, the proposition "A policeman is a government employee", is universal affirmative proposition but the proposition "A police man is at the gate", is particular affirmative proposition.

Device 7. Words like, "a few", "most", "majority", "many", "most of them", "several", "sometimes", "generally", "usually", "occasionally", "once", etc. suggest particularity of the propositions. For example:

"Many doctors are not greedy persons", is reduced into "O" proposition "Some doctors are not greedy persons".

"Most politicians are liars", is "I" proposition and its standard logical form is "Some politicians are liars".

Device 8. There are propositions in which all the components of the proposition such as subject term, predicate term, quality, quantity are present but they are not arranged in proper order. For instance :

"Tall buildings are all expensive".

"Insects are sometimes poisonous".

"Birds all have wings".

"Happy are those who know their limitations".

are translated as follows :

"All tall buildings are expensive things".

"Some insects are poisonous creatures".

"All birds are winged creatures".

"All those who know their limitations are happy beings" respectively.

Device 9. The propositions containing words like "except", "alone", "all but a few", "only some" are not simple propositions.

They are rather compound propositions. They mean two propositions simultaneously. Therefore, they are translated differently. Propositions like "All except children are allowed in the hall" means "All non-children (adults) are allowed in the hall" and "No child is allowed in the hall",[5] "Man alone is rational animal", is translated as the combination of two propositions. "All men are rational animals" and "No non-man is rational animal". The proposition "Only some students of this institution are Indian citizens", is translated as "Some students of this institution are Indian citizens" and "Some students of this institution are not Indian citizens".

Device 10. Sometimes a sentence has a negative predicate. It has to be translated in a proposition having a positive predicate. For example, the sentence "Some employees are non-residents", will be translated as "Some employees are not residents". Take another example "All fish are non-mammals", will be translated as "No fish is mammal". The negative predicate is changed into affirmative predicate by using obversion which is one of the important forms of immediate inference about which you will learn in chapter 6.

Device 11. Sometimes sentences are stated in interrogative form. But their meanings reveal that some of them are universal affirmative propositions while some others are universal negative propositions. For example sentences like:

1. "Is there a mother who does not love her children?"*
2. "Is there a man who is perfect?"

Are translated as :

1. "All mothers love their children."
2. "No man is perfect."

respectively.

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5. Cf. Irving M. Copi and Carl Cohen, op.cit., p. 283.

* This was suggested to me by my colleague Dr Rajib Roy, Reader, Philosophy Dept, K.M. College, University of Delhi, Delhi.

We have discussed various devices for translating the non-standard form propositions into standard logical form. They, however, are by no means exhaustive. There may be propositions which are not covered by any of the above devices, but in order to reduce such propositions into standard form, their meaning is the only guide.

Exercise 1

Reduce the following sentences into standard logical form propositions:

1. Every citizen is a voter.
2. All roses are not red.
3. The cuckoo is seldom seen.
4. Birds have wings.
5. Cats are mammals.
6. A businessman is hardly honest.
7. All old people are not wise.
8. Only mathematicians are good logicians.
9. All fish are non-mammals.
10. None but brave are winners.
11. Contractors are never dependable.
12. A few doctors are registered.
13. Few leaders are honest.
14. Good logicians are generally good mathematicians.
15. All children are non-voters.
16. Many politicians are not socialists.
17. Not even a single match was won by the official team.
18. Many birds build nests.
19. None but the graduates are applicants.
20. Children are rarely dishonest.
21. Policemen are not allowed in the religious institutions.

22. Every student of this class is a voter in the general election.
23. Only children are allowed to play in this park.
24. Boys are not allowed in this college.
25. Some good books are seldom read.
26. Usually children are emotional.
27. Mango is a delicious fruit.
28. A pen writes.
29. Soldiers are patriots.
30. Some residents are non-citizens.
31. Not all good speakers are good writers.
32. Most of the children speak Hindi in India.
33. Flowers are beautiful.
34. Only ladies are allowed in this bus.
35. Scientists are all genius.
36. Easy come, easy go.
37. Honesty is no where valued.
38. Under no circumstances, a lion will eat grass.
39. Selfishness is condemned everywhere.
40. Wars are sometimes necessary.
41. Every man cannot be a leader.
42. Any boy can swim in this swimming pool.
43. Only man is rational animal.
44. Psychopath can never be trusted.
45. Every dog has his day.
46. A few cars are expansive.
47. Few employees are hard working.
48. Fairies have wings.
49. Not all politicians are statesmen.
50. Artists usually are not rich.

Chapter 3 – Section B

Modern Logicians' Treatment of Categorical Propositions

THOUGH modern logicians[1] have accepted the traditional account of proposition in general, yet their treatment and classification of the propositions is different from that of the traditional thinkers.

The "existential import" of the propositions is the main cause of this difference. Before knowing the modern logicians' treatment and classification of the propositions, we must know about the existential import of the propositions.

Existential Import

Existential import means the commitment to the existence of a certain object or a thing, which is implied by the subject of a given proposition. If the class designated by the subject of the proposition has members, then the proposition is said to have existential import otherwise not. According to George Boole[2], a prominent modern logician, the universal propositions "A" and "E" do not have existential import while the particular propositions "I" and "O" have it.

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1. Modem logic originated in the first half of the ninteenth century. George Boole was one of its founder members. The other prominent logicians were B. Russell and A.N. Whitehead.
2. George Boole (1815–64) was a well-known British mathematician and logician. He, in 1840, established the technique of using mathematical symbols and operations to solve problems in logic. He devised a method of expressing logical relationships in terms of algebra, now known as "Boolean algebra".

Universal propositions "A" and "E" are existentially negative whereas particular propositions "I" and "O" are existentially affirmative. Universal propositions make no existential claim of entities but the particular propositions do make it. That is why, the particular propositions in modern logic are called existential propositions. Universal propositions differ from the particulars in respect of generality as well as in respect of existential import.

When I say "All men are mortal", I mean to accept that some relation holds between man and the characteristic of mortality. The proposition states that if anything is man, then it is mortal. I am not here accepting that there is an entity called man. I may presuppose or I may assume that there is such a thing called man. But the meaning of the proposition does not depend on the assumption that man exists. The meaning of the proposition depends on the acceptance of a relation between subject and predicate, and not on the assumption that the entity "man" exists. In the universal affirmative proposition what is accepted is only inseparable relation between the subject and the predicate term. The existence of the entity is merely assumed but not accepted.[3] What is accepted and what is assumed has to be carefully separated. Moreover, the existence of the object is neither necessary nor important for the meaningfulness of a universal proposition. For example, the proposition "All twentieth-century kings of France are bald" is perfectly meaningful in spite of the fact that there is no French king in twentieth-century. The proposition only states that if at all there is a French king in twentieth century, then he would be bald.

In the universal negative proposition also the existence of the object is merely assumed and not accepted. For instance, the proposition, "No man is immortal", states that if at all there is a "man", then it does not have the characteristic of immortality. The proposition is analyzed as, "Whatever X may be, if X is man, then he is not immortal". Both the universal propositions "A"

3. Cf. Peter Alexander, *An Introduction to Logic*, p. 79.

and "E" merely state the internal and inseparable relation between subject and predicate terms. The meaningfulness of these propositions ("A" and "E") does not depend on the existence of the objects indicated by the subject of the propositions but by the inseparable relation between subject and predicate terms.

But the situation is different in the particular propositions. The proposition "Some students are scholarship holders" is meaningful only when *there are* students among whom we differentiate scholarship holder and non-scholarship holder students. The meaningfulness of "I" or "O" proposition depends on the existence of the objects referred by the subject of the proposition. "Some students are scholarship holders" is analysed as "There exists at least one student who is scholarship holder". Similarly, "O" proposition "Some students are not hardworking" is analysed as, "There exists at least one such student who is not hardworking".

In the light of above discussion, it is important to examine when A, E, I and O propositions are true and when they are false. According to the traditional treatment of the propositions, a proposition is true when it describes the facts correctly, and false when it does not describe the facts correctly. But in the context of existential import, a proposition "All S is P" ("A") is true when there are no members in the class of subject term "S". Accordingly, "All S is P" is false when "S" (subject term) has members. Similarly, "No S is P" ("E") is true when the class of subject term "S" is empty and false when there are members in the class "S".

Particular propositions are true when there are members in the class of subject term. "Some S is P" is true when the class "S" has members, and false when "S" has no members. Similarly, "Some S is not P" is true when "S" has members and false when "S" is empty class.

In the following discussions you will see how the existential import of the propositions influenced George Boole's and John

Venn's[4] formulations of the categorical propositions. While Boole had formulated A, E, I and O propositions in terms *of empty classes,* Venn represented them through diagrams. But in their treatment of the categorical propositions, there is a presumption that universal propositions do not imply the existence of members in the subject term class whereas the particular propositions imply the existence of members in the subject term class. Aristotelian treatment of the propositions, however, presupposed that the classes represented by the terms of every categorical proposition have members.

Boolean Analysis of Categorical Propositions

Boolean interpretation of categorical propositions depends on the empty classes. A class is either empty or has members. Universal propositions are equated to empty class (zero) and particular propositions are not equal to empty class.

Proposition "All S is P" means no member of class "S" is outside the class "P". In other words the class having members of "S" but not of "P" is empty. The proposition "All S is P" is symbolized as:

$S\bar{P} = 0$

$\bar{P}$ means not P

"All scientists are philosophers" in Boolean formulation is expressed as:

$S\bar{P} = 0$

S - scientists
P- philosophers
$\bar{P}$ - non-philosophers

The proposition "All scientists are philosophers" means there is no scientist who is non-philosopher. Hence the class of non-philosopher scientists is empty.

4. John Venn (1834–1923) was a famous British logician who invented mechanical method of diagrams to express the categorical propositions.

Proposition "E", "No S is P" means there is nothing common between "S" and "P". In other words, there is not even a single member common between the classes of S and P.

$SP = 0$

"No socialists are pacifists", means there is not even a single socialist who is pacifist. Hence, it is expressed as

$SP = 0$

S = socialists
P = pacifists

"I" proposition "Some S is P" indicates there exists a class which has members of both the classes S and P.

$SP \neq 0$

"Some students are politicians", means there are students who are politicians also. This means there are members in the class of student politicians. Hence,

$SP \neq 0$

S - student
P - politician

The "O" proposition "Some S is not P", indicates there exists a class which has members of both S and non-P.

$S\bar{P} \neq 0$

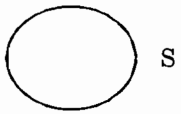
"Some students are not public figure", means there are students who are non-public figures. Hence, it is symbolized as :

S - student
P - public figure
$\bar{P}$ - non-public figure

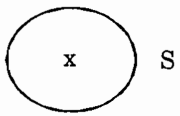
John Venn's Diagrams

There is yet another method of expressing categorical propositions in modern logic. John Venn represented the categorical propositions A, E, I and O through diagrams. Earlier Euler (1707-83), a Swiss mathematician, used diagrams in the areas of

calculus, geometry, algebra and number theory. Venn had also used method of diagrams to express categorical propositions. Later, the validity of arguments were tested by the methods provided by him.

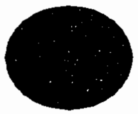


The circle represents class S



The circle shows the class S is not empty. It has members, say x

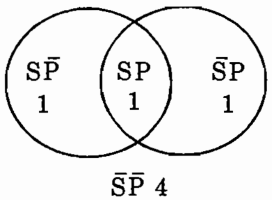
$S \neq 0$



The shaded circle shows the class S is empty. There is no member in the class S

$S = 0$

Venn diagrams are modification of Euler's diagrams. The key difference between Euler's diagrams and Venn's diagrams stems from the fact that Venn denied the existence of the objects in the universal propositions. The universal propositions for Venn, as in the case of George Boole, are existentially negative.

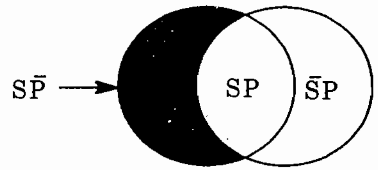


Number 1 segment of the circle represents that portion which has members of class S but not class P. Number 2 stands for the overlapping circles which has members of both the classes S and P. Number 3 represents that segment of circle which has members of P but not of S. Number 4 represents a class having members neither of S nor of P.

In each of the categorical propositions since there are two terms, subject and predicate, so there are two classes and two

circles. Let us see how the categorical propositions are diagrammed through circles.

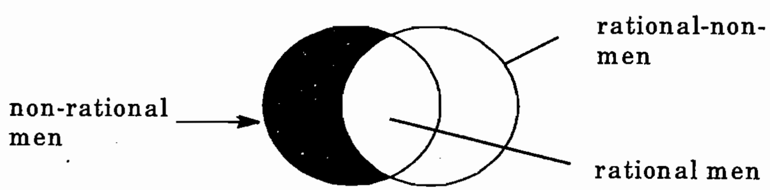
"All S is P", universal affirmative, "A" type proposition is shown through the diagrams as follows :



The shaded portion $S\bar{P}$ is shown empty. The proposition "All S is P" means no member of class S is outside the class P. Hence,

$S\bar{P} = 0$

For example: "All men are rational", is diagrammed as:



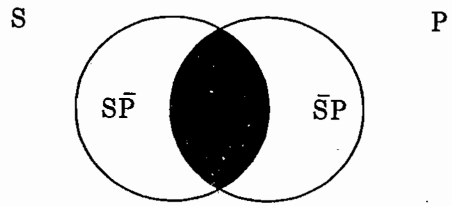
The class of non-rational man is empty.

$M\bar{R} = 0$

M - class of men

R - class of rationals

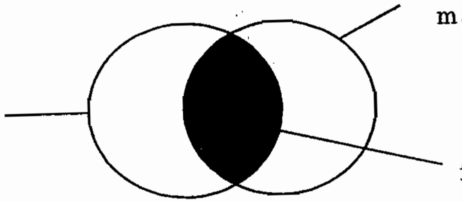
"E" proposition "No S is P", is diagrammed as:



"E" proposition states nothing is common between S and P. Both the classes are mutually exclusive. Hence,

$SP = 0$

"No fish is mammal", is diagrammed as:

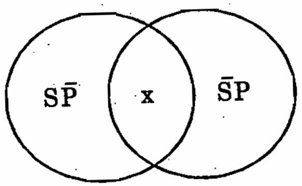


The class of fish mammal is empty.

$FM = 0$

F - fish
M - mammal

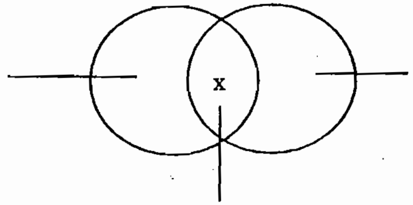
"I" proposition "Some S is P", is shown as:



"I" proposition shows there are common members in the classes of S and P. The class SP is not empty. It has members say x

$SP \neq 0$

"Some philosophers are religious leaders": is diagrammed as:



P - philosophers

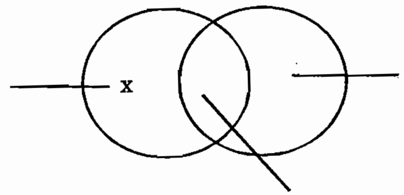
R - religious leaders

$PR \neq 0$

"O" proposition "Some S is not P", indicates there are members in the class S which do not share the characteristics of the class P.

$S\bar{P} \neq 0$

"Some animals are not carnivorous", is diagrammed as:



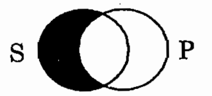
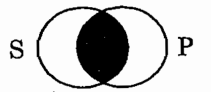
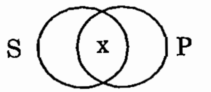
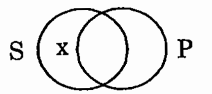
The proposition says there are non-carnivorous animals.

Hence, $A\bar{C} \neq O$

A - animals

C - carnivorous

Let us now compare all the three different formulations of the categorical propositions.

Proposition	*Traditional*	*Boolean*	*Venn*
1. Universal affirmative "A" type proposition	All S is P	$S\bar{P} = O$	
2. Universal negative "E" type proposition	No S is P	$SP = O$	
3. Particular affirmative "I" type proposition	Some S is P	$SP \neq O$	
4. Particular negative "O" type proposition	Some S is not P	$S\bar{P} \neq O$	

Exercise 2

Represent the following propositions through Venn diagrams:

1. All squares are polygons.
2. No egg is square.
3. Some cars are expensive items.
4. Some politicians are not sinners.
5. All learned are fortunate beings.
6. No circle is polygon.
7. Some communists are god fearing.
8. Some famous leaders are not degree holders.
9. All sailors are patriots.
10. No soldier is coward.

Modern Classification of Propositions

Modern logicians classified propositions as follows:

1. Categorical Propositions
   (i) Singular (Simple)
   (ii) General
2. Compound Propositions

Categorical Propositions

Both singular and general propositions are categorical propositions. A categorical proposition is singular when its subject term is one specific individual or object. A categorical proposition is general when the subject term represents either some or all the members of a class.

SINGULAR PROPOSITION

A singular proposition has a singular or a proper noun as its subject. The subject can also be uniquely referred to as individual object. For instance, "That chair is comfortable", "Ram is student of this college", "New Delhi is one of the crowded cities of India" are instances of singular propositions.

You have already seen earlier, the traditional logicians had never assigned any separate status to the singular propositions. They had included them under the universal propositions. But the modern logicians do not agree with them. They differentiate singular propositions from universal propositions because of the existential import. Whereas former are existentially loaded, the latter are existentially negative.

The singular propositions share the unrestricted generality (distribution) with universal propositions ("A" and "E") but not the existential import. Similarly the singular propositions share the existential import with the particular propositions ("I" and "O") but not their restricted generality (distribution). Therefore, the singular propositions are neither universal nor particular. They are unique and have separate status.

GENERAL PROPOSITION

The subject of a general proposition refers either to all the members of a class or to some members of a class. There are two types of generalizations, universal and particular, unrestricted and restricted. "A" and "E" are the universal propositions and "I" and "O" are particular propositions. Since you had already studied about them (they were discussed under the traditional

classification of propositions), we will not discuss them here any more.

Besides the difference of generality between universal and singular propositions, there is yet another major difference between the modern and the traditional logicians' treatment of the categorical propositions. As you have already seen the traditional logicians have recognized only **one form, "subject predicate form"** (the form which ascribes a predicate to a subject) for the categorical propositions. Propositions like:

(i) Gita is a hardworking student.

(ii) Gita is daughter of Mr X.

(iii) Gita is older than Mohan.

were treated as similar by them. They thought all the above propositions have one form, that is, subject predicate form. But the modern logicians differ. They are of the opinion that while the first proposition "Gita is hardworking student" has subject predicate form, the other two are relational propositions. The relational propositions can never be reduced to the "subject predicate form" propositions. The propositions "stating that two things have a certain relation have a different form from subject-predicate propositions, and the failure to perceive this difference or to allow for it has been the source of many errors in traditional metaphysics"[5]. The modern logicians thus extended the treatment of categorical propositions far beyond the traditional thinkers. Along with subject predicate form of the proposition, they recognized the relational form of categorical propositions also.

COMPOUND PROPOSITIONS

A compound proposition is constituted by two or more categorical propositions (singular or general). Basically three types of compound propositions are recognized. They are :

(i) Conjunctive

5. B. Russell, "Logic as the Essence of Philosophy" in *Essays in Logic*, ed. Ronald Jager, p. 127.

(ii) Disjunctive

(iii) Hypothetical

Conjunctive Proposition

Two categorical propositions when joined by "and", makes conjunctive compound proposition. For instance, "Ram is intelligent boy and he is hardworking". "She is dark but pretty".

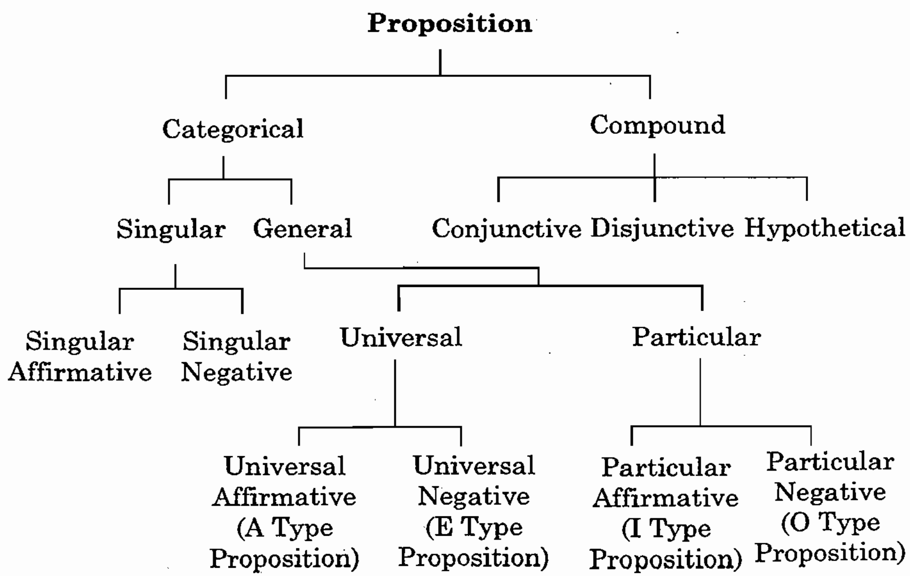
Disjunctive Proposition

Two categorical propositions when joined by the relation of "either or", then it is called disjunctive proposition. For instance, "Either he is sick or he is out of station", "Either it rains or we shall not go for picnic", etc.

Hypothetical Proposition

Two categorical propositions when joined by "if then" relation, then it produces a hypothetical proposition. For instance, "If I get admission, then I will complete the course in a year", etc.

Modern Classification of Proposition

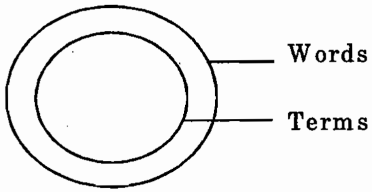


Chapter 4

Terms

A PROPOSITION is a statement in which something is said about something else either affirmatively or negatively. The thing about which something is said is subject of the proposition and that which is said about the subject is predicate. In the proposition "Ram is a hard working boy", "Ram" is subject term and "a hard working boy" is predicate term.

Both subject and predicate of a proposition are called terms. A term is a word or group of words which is either a subject or a predicate of a proposition. If a word or a group of words are neither a subject nor a predicate of a proposition, then it is not a term. In the propositions, "All men are mortal", "men" and "mortal" are terms but "all" and "are" are merely words for they are neither subject nor predicate of the proposition. Thus, whereas all terms are words, all words may not be terms.



A term is a word or group of words, but in addition to that it is:

(i) either a subject or a predicate of a proposition, and

(ii) it must have a definite meaning of its own.

The word "term" comes from the Latin word *terminus*, a limit or boundary. Terms limit the movement of the thought. Proposition is a unit of reasoning and terms are its constituents. In each categorical proposition A, E, I and O, there are two terms, subject and predicate.

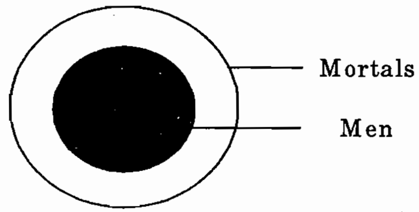
Three important aspects of a term are discussed here :

1. Distribution of terms
2. Denotation and Connotation of terms
3. Types of terms

Distribution of Terms

A term is said to be distributed if it refers to all the members of a class. For example, in the proposition, "All cats are mammals", the term "cat" is distributed for it refers to all the members of the class cat. If, on the other hand, a term refers to some or few members of a class, then it is called undistributed term. In the proposition, "Some cats are black", the term cat is undistributed for it refers to only part of the class cat. In other words, **a term is distributed when it includes or excludes all the members of a class,** and if a term includes or excludes only some members of a class, then it is undistributed.

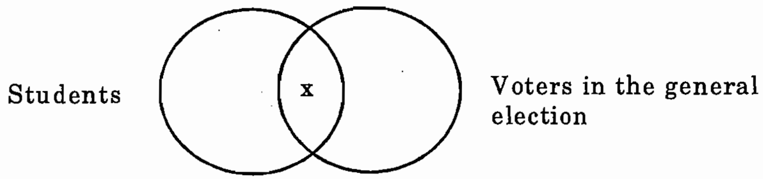
In a universal affirmative proposition, say "All S is P", subject term S is always distributed but predicate term P is undistributed. In the proposition "All men are mortal", the term "men" refers to all men and hence it is distributed, whereas the term "mortal" does not refer to all mortals (it refers to only some mortals, that is, human beings), hence it is undistributed. Look at the following diagram :



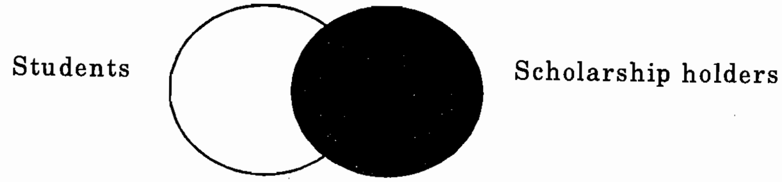
Whereas the class of men is in the preview of the class of mortals, the class of mortals is not in the preview of the class of men.

In a universal negative "E" proposition, "No S is P", both subject and predicate terms are distributed. In the proposition, "No men are immortal", all members of the class "men" are excluded from all members of the class "immortals". Therefore, both subject and predicate terms are distributed in "E" proposition.

In a particular affirmative, "I" proposition, say "Some S is P", none of the terms is distributed. In the proposition, "Some students are voters in the general election", the subject term "students" indicates "some" students, thus the subject term is undistributed. The predicate term "voters in the general election" also does not refer to all the members of the class of voters in the general election (there are voters in the general election who are not students). Thus, both subject and predicate terms are undistributed in "I" proposition.



In a particular negative, "O" proposition, "Some S is not P", subject term is undistributed but predicate term is distributed. "Some students are not scholarship holders" distributes its predicate term "scholarship holders" but not the subject term "students'. The subject term "students" here refers only to part of students, so the subject term in "O" proposition is undistributed but the predicate term "scholarship holders" is distributed. The meaning of the proposition states that the entire class of "scholarship holders" is excluded from some students. Hence, the predicate term is distributed in the "O" proposition.



The position of distributed and undistributed terms in the four categorical propositions is as follows:

Proposition	*Subject*	*Predicate*
A	Distributed	Undistributed
E	Distributed	Distributed
I	Undistributed	Undistributed
O	Undistributed	Distributed

The quantity of a proposition determines whether a subject term is distributed or not. If quantity of the proposition is universal as in "A" and "E" propositions, then subject term is distributed. If quantity of the proposition is particular as in "I" and "O" propositions, then the subject term is undistributed.

Quantity of a Proposition

Universal	Subject term distributed
Particular	Subject term undistributed

To determine whether predicate term is distributed or not, one looks to the quality of a proposition. In affirmative propositions ("A" and "I"), predicate term is always undistributed whereas in negative propositions ("E" and "O") predicate term is always distributed.

Quality of a Proposition

Affirmative	Predicate term undistributed
Negative	Predicate term distributed

The validity of deductive reasoning is partially decided by the distribution of terms. You will learn more about this in chapters 6 and 7.

Denotation and Connotation of Terms

The aim of a logician is to identify fallacies which arise in reasoning, and to suggest their remedies. Fallacies in reasoning may arise from various sources. The wrong use of language is one of them. Sometimes the meaning of terms are not known or merely half known. In that case their use in reasoning may make it fallacious. In order to keep our reasoning straight, the meaning of the terms used in the reasoning should be absolutely clear. The ambiguous or vague terms must not be used in the propositions. The logicians have suggested two different techniques for making the meanings of terms clear. They are, denotative and connotative techniques. Let us first see what is denotation and what is connotation of a term.

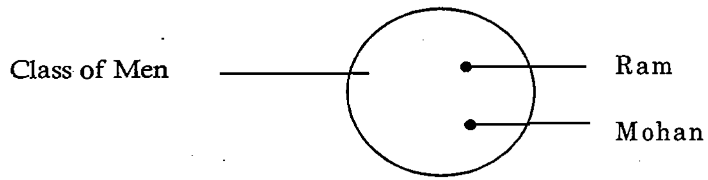
Denotation of a term is the object or thing referred by the term. For example, the denotation of the term "man" is Gita, Mohan, Kamala, etc. The denotation of the term "poet" is Wordsworth, Keats, Mahadevi Verma, etc.

Connotation of a term is the set of qualities and characteristics possessed by the objects or things referred by the term. For example, the connotation of the term "man" is mortality and rationality. Connotation means those common characteristics which are possessed by all the members denoted by the term. Mortality and rationality are two characteristics possessed by all men. Other characteristics like being educated, Indian, employed, tall, etc. are not possessed by every man and, therefore, are not essential. So the connotation of a term refers only to those qualities which are minimum but absolutely indispensable for the meaningfulness of a term.

Denotation of a term refers to the objects or things which possess the qualities. Connotation refers to the set of characteristics essentially possessed by every object denoted by the term. Denotation denotes the objects, connotation connotes the characteristics.

The concept of denotation and connotation can be made clearer in the context of "class'. The class of "man" for instance, has two-

fold aspects. The members belonging to the class say, x, y, z, etc. are denotation of the term "man'. The class characteristics "mortality" and "rationality" possessed by every member of the class man, is the connotation of the term "man'. The members of the class are the denotation and characteristics possessed by them is connotation.



Hence, a class carries twofold meaning, denotative and connotative. Denotative meaning of a term is referential or extensional for it refers to members of the class. Denotation fixes the boundary of the term and hence, indicates the extent to which the term is applicable.

Connotative meaning of a term is intentional which means the intended and indicated characteristics of a term.

Generally three types of connotations[1] are recognized. They are

(i) Subjective

(ii) Objective

(iii) Conventional

A logician, however, is interested only in conventional connotation. Neither subjective nor objective connotation is of any use to him.

(i) **Subjective Connotation**: Sometimes a term refers to a set of characteristics which differs from one group of people to another. In that case there is no one common or fixed connotation of the term. For example, the term, "married woman in white clothes" connotes a figure of

1. Cf. Irving M. Copi, *Introduction to Logic* (sixth edition), p. 155.

sorry woman in India (widowhood) whereas the same term connotes a figure of happy woman in Western countries (wedding dress). Such connotations are subjective and are of no use to a logician who is interested in that kind of connotation which is relatively fixed and common for all.

(ii) **Objective Connotation**: The connotation of a term when it refers to all the characteristics, known and unknown, then it is called objective connotation. A human being, however, can never be sure whether the characteristics he knows are all or merely few. Only an omniscient or super being can claim to know all the characteristics referred by a term. We know the term "man" has two characteristics, mortality and rationality, and the other characteristics of the term "man", if at all they are there, are still not known to us. Therefore, a logician is not interested in objective connotation either. He is interested only in the conventional connotation.

(iii) **Conventional Connotation**: The conventional connotation of a term is the set of all the known common characteristics possessed by all the members denoted by the term. Accepted and agreed set of qualities of a term saves us from the defects of subjective as well as objective connotations. In a way conventional connotation is a middle path between them. It is not wrong to say that conventional connotation is a definition of a term, and definition is common for all. If a set of characteristics of a term changes, so also definition of the term.

RELATION BETWEEN CONNOTATION AND DENOTATION

Though theoretically connotation and denotation of a term can be discussed separately yet they are inseparably related. The relation between them is of inverse variable which means if connotation increases, denotation decreases and if connotation decreases, denotation increases. An example is illustrated here for the better understanding of this relationship.

Connotation	*Corresponding denotation*
(i) Mortality	All living beings (men, animals and plants)
(ii) Mortality + Rationality	Only human beings
(iii) Mortality + Rationality + Indian	Indian men and women
(iv) Mortality + Rationality + Indian + Educated	Only educated Indians are members of the new class

The above illustration shows that with the increase in qualities (connotation), the members (denotation) decreases. Similarly, with the decrease in the connotation, denotation increases. The lesser the qualities, more are the members referred by the term.

However, it is the connotation which determines denotation, and not the other way round. The set of characteristics fixes the members in a class. The denotation, the things or the objects do not fix the qualities very satisfactorily. It is not appropriate to say that with the increase in denotation, connotation decreases always or with the decrease in denotation connotation increases. The trouble may arise in those situations where with the increase of denotation we expect connotation to decrease or with the decrease in denotation we expect connotation to increase

Let us illustrate this.

With the increase in man's population can we say the connotation, the essential characteristics of the term "man", mortality and rationality have decreased? Certainly not. The connotation of the term "man" (rationality and mortality) remains the same. Thus, with the increase or decrease of denotation, the connotation either remains where it is or if at all it (connotation) changes, it will change in opposite direction.

Though both connotation and denotation of a term are inseparable aspects of a term, yet it is connotation which is more basic and fundamental. Through connotation, it is easier to locate

appropriate denotation. But by starting with denotation, connotation is not so accurately determined.

There are terms which have both connotation and denotation. General terms and common nouns are examples of such terms, and they are called "Connotative Terms".

However, some terms, for example, singular terms and proper nouns have proper references but not proper connotation. In such cases connotation is either incomplete or ambiguous. For example, the term "Ram" has a suitable reference (denotation) but the meaning of the term "Ram" may mean a student who is my friend, or a legendary figure in Indian mythology or an employee. The connotation of the term "Ram" is ambiguous in the sense that it can be used differently in different context. Unless further information is provided, the meaning of the term "Ram" is merely half. The singular terms are like variables. They keep changing their values according to the context in which they occur. "The singular term 'Jones' is ambiguous in that it might be used in different contexts to name any of various persons. . ."[2].

But there are some terms which have proper connotation but not proper denotation. For instance, the term "square round" (being a non-existent reality) does not have reference, though the term has definite connotation. There are null classes, empty classes, where denotation is Zero.

The terms which do not have both connotation and denotation (either proper connotation is missing or a proper denotation is not there) are called "non-connotative" terms by J.S. Mill. Common nouns and general terms are connotative terms whereas proper nouns, abstract terms, singular terms are non-connotative.

Types of Terms

Logicians have classified terms into various groups. Broadly terms are classified as :

2. W.V. Quine, *Methods of Logic* (third edition), p. 217.

(i) Singular and General Terms
(ii) Concrete and Abstract Terms
(iii) Positive and Negative Terms
(iv) Collective and Distributive Terms

SINGULAR AND GENERAL TERMS

A singular term refers to one single, or specific person, object or thing. For instance, Socrates, New Delhi, this book, that man, etc. According to Quine, "a term is singular if it purports to name an object (one and only one), and otherwise general"[3]. "I" and "thou" are also singular terms. A general term, on the other hand, is used as common noun. For instance, in the proposition "Man is mortal", the term "man" is used as common or general term. All common nouns when used as terms in propositions are general terms. For instance, school, student, college, pen, cat, etc. are general terms. In logic general terms are also known as classes.

Singular terms are of two types :

(i) Proper names or nouns
(ii) Specifically or uniquely described term.

Singular terms which are proper nouns can be easily recognized. All proper nouns or names when used in propositions either as a subject or as a predicate are called singular terms, for example, India, Sita, University of Delhi, etc.

Specifically and uniquely descriptive terms are those which though not proper names yet have a unique reference. Uniquely descriptive term refers to single person or object. For example, "Women Prime Minister of India up to 1982" has a unique reference; it refers to a single being Mrs Indira Gandhi. Hence the term "Woman Prime Minister of India up to 1982" is a singular term.

3. W.V. Quine, *Methods of Logic* (third edition), p. 218.

CONCRETE AND ABSTRACT

Terms that denote things, objects, persons and articles are concrete, for example, chair, pen, book, crow, man, etc. Abstract terms, on the other hand, refer to abstract entities, characteristics or attributes in themselves. "Concrete terms are those which purport to refer to individuals, physical objects, events; abstract terms are those which purport to refer to abstract objects, e.g. to numbers, classes, attributes"[4]. For instance, happiness is an abstract term but happy is not abstract term. An abstract term is not simply the name of a quality but a quality considered by itself. Adjectives are not abstract terms because an adjective refers to quality as qualifying objects. For example, in the proposition "Trees are green", "green" is not an abstract term for it refers to quality of greenness as qualifying trees. So "greenness" is abstract term whereas" green" is merely an adjective. It should be recalled that both subject and predicate terms are nouns. If either of the terms is not noun, then it is to be changed into appropriate noun. The above proposition. "Trees are green", is reduced to "Trees are green objects". Both subject term "trees" and predicate term "green objects" are concrete terms.

J.S. Mill points out that concrete terms and abstract terms usually go in pairs. For example, happy — happiness, triangle — triangularity, man man — hood, strong — strength, etc. However, the distinction between concrete and abstract term is not absolute. The same term can be used as concrete term in one context and abstract in another context. For example, in the proposition, "Cricket team got hostile" the term "cricket" is used as concrete term whereas in the proposition "Cricket team teaches discipline" the term "cricket" is used as an abstract term. Similarly in the proposition, "I have faith in honesty", the term "honesty" is used as concrete term, whereas in the proposition, "Honesty is virtue" the term "honesty" is used as an abstract term.

4. W.V. Quine, *Methods of Logic* (third edition), p. 217.

POSITIVE AND NEGATIVE TERMS

A positive term is one which posit or affirm the presence of an attribute. For instance, wise, true, happy, voters, citizens, etc. are positive terms. Negative terms deny the presence of an attribute like unwise, unhappy, immortal, bald, orphan, etc. Negative terms are not to be confused with negative propositions. For example, "Some men are not wise" is a negative proposition, whereas "Some men are unwise" is affirmative proposition with a negative predicate.

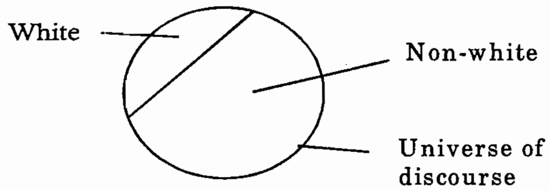
The distinction between positive and negative term becomes extremely important in the context of contradictory and contrary terms.

Two terms are contradictory if they are mutually exclusive and collectively exhaustive. For instance, equal and unequal pair of terms exhaust the entire class of relation and, hence are contradictory. The universe of discourse (the field of reference) is exhausted by them. Equal or unequal, black and non-black, white and non-white are examples of contradictory terms.

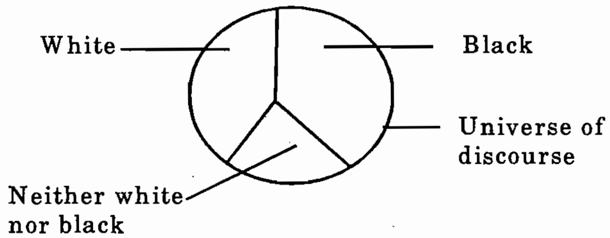
One easy way to make pair of contradictory terms is by prefixing "non" or "not" to a term such as happy and non-happy, black and non-black, Indian and non-Indian, voters and non-voters. It is very difficult to find pair of contradictory terms in our practical life. In the formal sciences, however, they are easily available.

Contrary terms just as contradictory terms cannot coexist and are incompatible, yet contrary terms do not exhaust entire universe of discourse. **A pair of terms are contrary when they are mutually exclusive but not exhaustive.** Black and white colours are exclusive but do not exhaust the field of colours. There are colours which are neither black nor white. It could be red, blue, green or yellow. Black is not white colour but both black and white do not cover or exhaust the entire field of colours.

Contradictory Terms
(White, non-white)



Contrary Terms
(White is contrary to black colour)



COLLECTIVE AND DISTRIBUTIVE TERMS

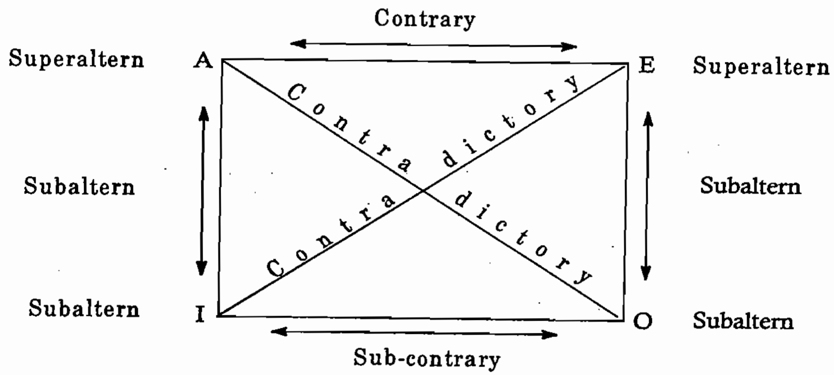
Collective terms apply to group of persons, things or objects but not to the individuals composing the group. For instance, terms like club, parliament, society, library, army, etc. are collective terms. The use of collective term means the entire class is taken into consideration. On the other hand, distributive term refers to all the members of the class separately. In the following propositions there is a use of distributive terms, "All plays of Shakespeare are outstanding pieces of literature", "All members of Indian army are Indian citizens", "All members of the club are Citibank card holders". However, no term by itself is collective or distributive. The use of the term in a proposition decides whether a term is collective or distributive. The collective use of the term takes term as a whole whereas use of distributive term takes into consideration all the members of the class individually.

Chapter 5

Square of Opposition
(Relation Among the Categorical Propositions)

FOUR categorical propositions A, E, I and O are related to each other by various "truth relations". The relations among them are of "opposition". The traditional logicians have explained these relationships by a diagram of square to which they call "Square of Opposition".

Each side of the square and also the diagonals represent different kinds of relations.



Universal affirmative "A" proposition, "All S is P" and Universal negative "E" proposition, "No S is P" are related to each other by the Contrary relation. The proposition "All scientists are philosophers" is contrary to "No scientists are philosophers". Similarly, "No fish is mammal" is contrary to "All fish are mammals".

Particular affirmative "I" proposition, "Some students are scholarship holders" is related to "O" proposition "Some students are not scholarship holders" by subcontrary relation. Similarly, "O" proposition, "Some animals are not carnivorous" is related to "I" proposition, "Some animals are carnivorous" by subcontrary relation.

The Universal affirmative "A" proposition, "All S is P" is related to particular negative "O" proposition, "Some S is not P" by contradictory relation. The contradictory of "All men are mortal" is "Some men are not mortal" and vice versa. The contradictory of "E" proposition, "No crows are mammals" is "I" proposition, "Some crows are mammals".

"Universal affirmative "A" proposition, "All men are mortal" is superaltern to "I" proposition, "Some men are mortal". Similarly, "E" proposition, "No crow is mammal" is superaltern to "O" proposition, "Some crows are not mammals".

1. "All S is P" is *contrary* to "No S is P" and vice versa.
2. "Some S is P" is *subcontrary* to "Some S is not P" and vice versa.
3. (i) "All S is P" is *contradictory* to "Some S is not P" and vice versa.
   (ii) "No S is P" is *contradictory* to "Some S is P" and vice versa.
4. (i) "All S is P" is *superaltern* to "Some S is P".
   (ii) "No S is P" is *superaltern* to "Some S is not P".
   (iii) "Some S is P" is *subaltern* to "All S is P".
   (iv) "Some S is not P" is *subaltern* to "No S is P".

The relationship among the categorical propositions is of opposites. Two categorical propositions are said to be opposite if they differ either in:

i. quantity, or
ii. quality, or
iii. both quantity and quality.

Contrary propositions "A" and "E" have same quantity (universal) but they differ in qualities. Similarly, subcontrary propositions "I" and "O" have same quantity (particular) but they differ in qualities. Propositions related by subaltern relationship such as "A" and "I", and also "E" and "O" have the same quality but they differ in quantities. Contradictory propositions "A" and "O" and also "E" and "I", however, differ both in quantities and qualities.

Each opposite relation has certain characteristics. Contrary proposition "A" and "E" *cannot be both true together* though they both can be false at the same time. If one of the contrary propositions is true then the other contrary proposition is necessarily false, whereas if one contrary proposition is false, the other contrary proposition is undetermined (it can be true or false).

Subcontrary propositions "I" and "O" *cannot both be false together* though they both can be true together. If "I" is true, "O" is undetermined; whereas if "I" proposition is false, "O" is necessarily true. Similarly, if "O" is true, "I" is undetermined but if "O" is false, "I" is definitely true.

Subaltern relationship shows if "A" is true, then "I" is necessarily true, but if "I" is true, "A" remains undetermined. Same is the case with "E" and "O". If "E" is true, "O" is true but not the other way round.

Contradictory relation between "A" and "O", and also between "E" and "I" is of strict opposition. If "A" is true, "O" is false; if "O" is true, "A" is false. Similarly, if "E" is true, "I" is false and if "I" is true, "E" is false.

These various relations lay down the foundation for testing the validity of some elementary arguments in logic as Irving M. Copi says, "The relationships diagrammed by this Square of Opposition were believed to provide a logical basis for validating

certain elementary forms of arguments".[1] The "Square of Opposition", thus, is form of inference, immediate inference. The inference can be immediate or mediate. In the immediate inference, the conclusion follows from a single premiss. In other words, just one premiss is enough to imply the conclusion in such type of inference. From "a single proposition expressed in one or other of the four standard forms, it is possible to infer, directly from the given proposition other propositions".[2]

If "A" proposition is given as true, then it implies:

E proposition is false

I proposition is true

O proposition is false

If "A" proposition is given as false, then it implies :

E proposition is undetermined

I proposition is undetermined

O proposition is true

If "E" proposition is given as true, then it implies :

A proposition is false

I proposition is false

O proposition is true

If "E" proposition is given as false, then it implies :

A proposition is undetermined

O proposition is undetermined

I proposition is true

If "I" proposition is given as true, then it implies:

A proposition is undetermined

O proposition is undetermined

E proposition is false

1. Irving M. Copi, and Cohen Carl, *Introduction to Logic* (ninth edition), p. 217.
2. S.H. Mellone, *Elements of Modern Logic*, p. 84.

If "I" proposition is given as false, then it implies:

A proposition is false

O proposition is true

E proposition is true

If "O" proposition is given as true, then it implies:

A proposition is false

E proposition is undetermined

I proposition is undetermined

If "O" proposition is given as false, then it implies:

E proposition is false

A proposition is true

I proposition is true.

In "Square of Opposition", one important point should be noticed, that is in order to find out the other opposite relations from a given proposition, **the subject and predicate of the implied proposition is same as that of the implying one.**

Here are some solved examples :

Example 1 : The contrary contradictory and subaltern of the proposition "All men are mortal" are as follows :

Contrary	"No men are mortal" (E)
Contradictory	"Some men are not mortal" (O)
Subaltern	"Some men are mortal"(I)

Example 2 : The subcontrary, contradictory and superaltern of the proposition "Some scientists are mathematicians" are as follows :

Subcontrary	"Some scientists are not mathematicians" (O)
Contradictory	"No scientists are mathematicians" (E)
Superaltern	"All scientists are mathematicians" (A)

Modern Logicians "Square of Opposition"[3]

The traditional "Square of Opposition" was criticized by the modern thinkers in the context of the existential import. You have already read in chapter 3B that universal propositions, "A" and "E" do not have existential import whereas the particular propositions "I" and "O" do have it. This standpoint had a tremendous impact on the traditional "Square of Opposition". Consequently, only one relation of contradictory among the propositions has been accepted as valid. Other relations like contrary, sub-contrary, subaltern have become invalid.

The existential import of the categorical propositions states that the propositions "All S is P" is true when the subject term "S" has no member. In other words, when "S" is an empty class, then "All S is P" as well as "No S is P" are true propositions. However, if the subject term "S" has members then "All S is P" and also "No S is P" are false.

But the proposition "Some S is P" is true when the subject term "S" has members; whereas, if "S" has no member, then the proposition "Some S is P" is false. The same is true for the "O" proposition "Some S is not P". The proposition, "Some S is not P" is true when the subject term "S" has membes and false when the subject term "S" has no member.

In this context, let us examine the various relations of "Square of Opposition".

Let us begin with contrary relation. The traditional account of the contrary relation states that propositions "All S is P" and "No S is P" (which are related by contrary relations) cannot be both true together though they both can be false together. But the propositions, "All houses made of cheese are green houses" and "No houses made of cheese are green houses" are true together because the subject term of these propositions "houses made of cheese" is an empty class (there is no member in the

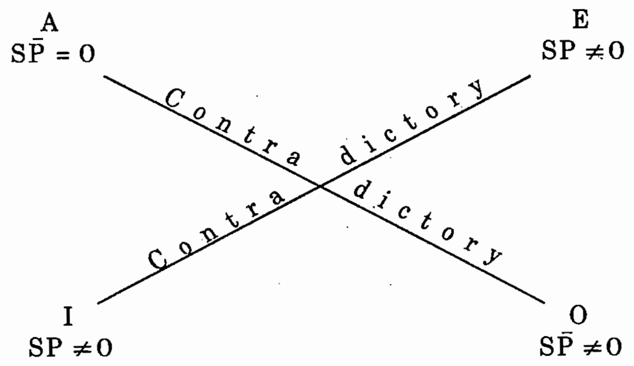
3. Here I am taking mainly Boolean interpretation of categorical propositions.

class of "houses made of cheese"), and you already know "A" and "E" propositions are true when their subject term is an empty class. Now if both "A" and "E" propositions are true together, then how can they be contraries?

The subcontrary relation between "I" and "O" propositions also becomes invalid. The existential import maintains "I", "Some S is P" and "O", "Some S is not P", are true when the subject term "S" has members; and the propositions "I" and "O" are false when the subject term "S" has no members. The traditional account of the subcontrary relation is that though both "I" and "O" propositions can be true together, they cannot be false together. But if there are no member in the subject term of "I" and "O" propositions, then both these propositions are false together. For instance, "Some high golden mountains are found in India" and "Some high golden mountains are not found in India" are false together since the subject term "high golden mountains" is an empty class. Now if both "I" and "O" are false together, then they are no more subcontraries.

Not only that, the subaltern relation between "A" and "I" propositions, and between "E" and "O" also become invalid. According to the traditional account of subaltern relation, the truth of "A" proposition implies the truth of "I" proposition and similarly, the truth of "E" proposition implies the truth of "O" proposition. But if one accepts the existential import, then from the truth of "A" proposition, the truth of "I" cannot be validly inferred. Similarly, the truth of "E" proposition does not imply the truth of "O" proposition. The reason is, "A" and "E" propositions being existentially negative propositions cannot imply the truth of "I" and "O" propositions respectively because they are existentially loaded. In the deductive reasoning, the conclusion cannot possess anything new or extra which the premisses do not have. The premisses "A" and "E" do not have existential import and thus cannot imply the truth of "I" and "O" propositions respectively which have existential import.

Modern "Square of Opposition"



Now you can imagine the big blow the traditional "Square of Opposition" had to suffer because of existential import. The new and modern "Square of Opposition" accepts only one relation of contradiction.

Exercise 3

A. Find contrary/subcontrary, contradictory and subaltern/ superaltern of the following propositions:

1. All voters are citizens.
2. No philosophers are traders.
3. Some leaves are fragrant.
4. Some good speakers are not good writers.
5. All cats are mammals.
6. No dishonest person is brave.
7. Some politicians are non-statesmen.
8. All non-mortals are non-men.
9. Some non-graduates are not non-citizens.
10. No artists are wealthy persons.

B. If "All butterflies are beautiful creatures" is given as true, what can be inferred about the following:

1. No butterflies are beautiful creatures.
2. Some butterflies are beautiful creatures.
3. Some butterflies are not beautiful creatures.

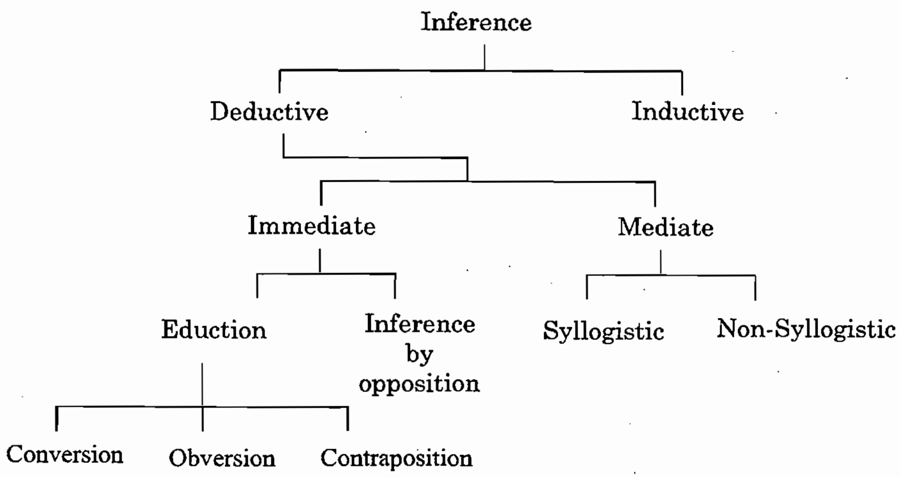
C. If "Some lions are black" is given as false, what can be inferred about the following :

1. All lions are black
2. No lions are black
3. Some lions are not black.

Chapter 6

Immediate Inference
(Eduction)

INFERENCE means to draw a conclusion from a premiss or premisses. Inference, however, is of different types. Traditionally it is classified as follows:



The basis of classification of immediate and mediate inference is the number of premisses in the reasoning. In immediate inference there is one and only one premiss whereas in mediate inference there are at least two premisses, and jointly they imply the conclusion. An argument having two and only two premisses is called syllogistic mediate inference. Aristotle is known for his invention of the theory of syllogism.

If the number of premisses are more than two, then the mediate reasoning becomes non-syllogistic.

But every deductive reasoning (immediate or mediate) follows the rule of distribution of terms. The rule states that **if a term is distributed in the conclusion, then it must necessarily be distributed in the premiss.** A piece of deductive reasoning that does not obey this rule becomes invalid. In no valid deductive reasoning conclusion is wider than the premisses.

Immediate inference : In immediate inference there is one and only one premiss and from this sole premiss conclusion is drawn. Immediate inference is of two types :

(i) Inference by opposition

(ii) Eduction.

You have already studied inference by opposition in the previous chapter. In the present chapter, you will study various forms of eduction. In eduction the meaning of the premiss and the conclusion is same. The difference between them is only of forms. But in inference by opposition ("Square of Opposition") the premiss and the conclusion differ both in form and meaning.

Eduction

Eduction is of three types:

(i) Conversion

(ii) Obversion

(iii) Contraposition.

Conversion and obversion are independent forms of immediate inference, whereas contraposition is dependent on them.

CONVERSION

In conversion the following rules are observed :

1. The subject of the premiss becomes predicate of the conclusion and predicate of the premiss becomes subject of the conclusion.
2. Quality of the premiss and of the conclusion remains the same. If the premiss is affirmative, then the conclusion is

also affirmative and if the premiss is negative, the conclusion is also negative.

3. Quantity of the premiss and the conclusion should be same as far as possible. If the premiss is universal, then the conclusion should also be universal. Similarly, if the premiss is particular, then the conclusion should also be particular. However, in the conversion of A proposition this rule is not observed.
4. The rule governing distribution of terms must be observed.

Conversion of the four categorical propositions A, E, I and O is as follows:

Conversion of A Proposition

Keeping first three rules in mind, the proposition "All S is P" is converted as :

All S is P.

Therefore, all P is S.

But it is invalid reasoning; for the term P which is distributed in the conclusion fails to be distributed in the premise. We can try an alternative conclusion.

All S is P.

Therefore, some P is S.

This is valid. While proposition A could not be converted into A, it can be converted into I proposition. This conversion of A into I, however, is called *conversion by limitation* because here we go from universal premiss to particular conclusion.

"All men are mortal" is converted as "Some mortals are men".

Conversion of E Proposition

No S is P.

Therefore, no P is S.

is valid reasoning, and hence the conversion of E is into E.

"No applicants are graduates" is converted into
"No graduates are applicants".

Conversion of I Proposition

Some S is P.

Therefore, some P is S

is valid.

"Some graduates are employed" is converted as

"Some employed persons are graduates."

Conversion of O Proposition

The conversion of O proposition is not valid. Let us see why.

Some S is not P.

Therefore, some P is not S

is invalid, for the term S which is distributed in conclusion fails to be distributed in the premiss. So, no conversion is possible for O proposition.

Summary

A is converted into I.
E is converted into E.
I is converted into I.
Conversion of O proposition is not valid.

OBVERSION

Obversion is another independent form of immediate inference. The rules for obversion are as follows :

1. Subject of the premiss is the subject of the conclusion.
2. Predicate of the conclusion is contradictory of the predicate of the premiss. One must notice that a negative proposition is different from the proposition having a contradictory predicate. For instance, "Some S is not P" is a negative proposition whereas "Some S is non-P" is affirmative proposition with contradictory predicate.
3. Quantities of the premiss and the conclusion must be same. If the premiss is universal, conclusion is also universal and if the premiss is particular, conclusion is also particular.

4. Qualities of the premiss and the conclusion, however, are different. If the premiss is affirmative, then the conclusion is negative; and if the premiss is negative, the conclusion is affirmative.
5. The rule of distribution of the terms is to be observed.

Obversion of A Proposition

All S is P.
______
Therefore, no S is non-P

is valid for it satisfies all the rules of obversion.

"All men are mortal" is obverted as
"No men are immortal".

Obversion of E Proposition

No S is P.
______
Therefore, all S is non-P

is valid inference

"No politicians are idealists" is obverted as
"All politicians are non-idealists".

Obversion of I Proposition

Some S is P.
______
Therefore, some S is not non-P

is valid inference.

"Some men are wise" is obverted as
"Some men are not unwise".

Obversion of O Proposition

Some S is not P.
______
Therefore, some S is non-P

is valid inference.

"Some applicants are not Indian citizens" is obverted as
"Some applicants are non-Indian citizens".

Summary

A is obverted into E.
E is obverted into A.
I is obverted into O.
O is obverted into I.

CONTRAPOSITION

Contraposition is not an independent form of immediate inference. It is a combination of both conversion and obversion. In contraposition, the subject of the conclusion is contradictory of the predicate of the premiss, and predicate of the conclusion is contradictory of the subject of the premiss. We keep applying obversion and conversion by turns till the required subject and predicate is found.

Contraposition of A Proposition

	All S is P	
=	No S is non-P	by obversion
=	No non-P is S	by conversion
=	All non-P is non-S	by obversion

The required subject and the required predicate is found. Therefore, the contraposition of A proposition is A itself.

"All cats are mammals" is contraposited as "All non-mammals are non-cats".

Contraposition of E Proposition

	No S is P	
=	All S is non-P	by obversion
=	Some non-P is S	by conversion
=	Some non-P is not non-S	by obversion

The required subject and the required predicate are found, therefore, contraposition of E proposition is O.

"No businessmen are philosophers" is contraposited as "Some non-philosophers are not non-businessmen".

Since E is contraposited into O, it is called *contraposition by limitation* because the premiss is universal whereas the conclusion is particular.

Contraposition of I Proposition

Some S is P

= Some S is not non-P — by obversion

Now is turn for conversion but since it is O proposition, its conversion is not possible.

Therefore, there is no valid contraposition of I proposition.

Contraposition of O Proposition

Some S is not P

= Some S is non-P — by obversion

= Some non-P is S — by conversion

= Some non-P is not non-S — by obversion

The required subject and the required predicate are found, therefore, contraposition of O proposition is O itself.

"Some philosophers are not radicals" is contraposited as "Some non-radicals are not non-philosophers."

Summary

A "All citizens are voters" is contraposited into "All non-voters are non-citizens".

E "No politicians are honest" is contraposited into "Some non-honest beings are not non-politicians".

I "Some applicants are graduates" cannot be contraposited.

O "Some students are not voters" is contraposited as "Some non-voters are not non-students".

Various forms of eduction of the categorical propositions are tabled as follows:

Proposition	*Conversion*	*Obversion*	*Contraposition*
All S is P	Some P is S (conversion by limitation)	No S is non-P	All non-P is non-S
No S is P	No P is S	All S is non-P	Some non-P is not non-S (contraposition by limitation)
Some S is P	Some P is S	Some S is not non-P	Not Valid
Some S is not P	Not Valid	Some S is non-P	Some non-P is not non-S

It should be noticed that in all the forms of eduction, the premiss and the conclusion are equivalent in meanings though the forms of the premiss and of the conclusion are different. Hence, conversion obversion, and contraposition are called equivalent propositions also. Each categorical proposition has at least two equivalent propositions.

The proposition "All S is P" is equivalent to :

(i) All S is P = No S is non-P — Obversion

(ii) All S is P = All non-P is non-S — Contraposition

Conversion of A into I is not equivalent proposition. For, one is universal and the other is particular.

The proposition "No S is P" is equivalent to :

(i) No S is P = No P is S — Conversion

(ii) No S is P = All S is non-P — Obversion

Contraposition of E proposition into O are not equivalents for one proposition is universal and the other is particular.

The I proposition "Some S is P" has again two equivalents :

(i) Some S is P = Some P is S — Conversion

(ii) Some S is P = Some S is not non-P — Obversion

Contraposition of I proposition is not valid.

Equivalents of O proposition are:

(i)	Some S is not P = Some S is non-P	Obversion
(ii)	Some S is not P = Some non-P is not non-S	Contraposition

Conversion of O proposition is not valid.

Note: In order to find out conversion, obversion and contraposition of a categorical proposition, one must make sure that the proposition is in standard logical form. There must be one subject term (noun), one predicate term (noun) and they must be joined by "is", "is not", "was", "was not", "are", "are not", etc. If predicate term of a proposition is not a noun, then it should first be changed into substantive noun and then the forms of eduction are to be applied.

Here are some solved examples:

(1) Find conversion, obversion and contraposition of the following propositions:

(i) All cats eat meat.

The proposition is not in the standard logical form. After changing it into standard form it becomes

All cats are meat eaters.

Conversion	Some meat eaters are cats.
Obversion	No cats are non-meat eaters.
Contraposition	All non-meat eaters are non-cats.

(ii) All boys play football.

Logical form of the proposition is:

All boys are football players.

Conversion	Some football players are boys.
Obversion	No boys are non-football players.
Contraposition	All non-football players are non-boys.

(iii) No politicians are honest.

Logical Form	No politicians are honest beings.

Conversion	No honest beings are politicians.
Obversion	All politicians are dishonest beings.
Contraposition	Some dishonest beings are not non-politicians.

(2) Find conversion, obversion, contraposition, contrary/ subcontrary and contradictory of the following propositions:

(i) Some students are non-radicals.

Conversion	Some non-radicals are students.
Obversion	Some students are not radicals.
Contraposition	Since it is I proposition, its contraposition is not valid.
Subcontrary	Some students are not non-radicals.
Contradictory	No students are non-radicals.

(ii) Some artists are not non-professionals.

Conversion	Since it is O proposition, its conversion is not valid.
Obversion	Some artists are professionals.
Contraposition	Some professionals are not non-artists.
Subcontrary	Some artists are non-professionals.
Contradictory	All artists are non-professionals.

(3) A few more examples:

(i) If "Some students are scholarship holders" is true, what can be said about "All students are non-scholarship holders"?

Note : In such types of problems, first change the subject and the predicate of the *given proposition* so that they match the subject and the predicate of the *required proposition* (the proposition whose value you want to find). This can be done by applying either conversion or obversion or contraposition as the situation demands. The forms of eduction can be applied repeatedly also. After that if need arises, apply "Square of Opposition".

Solution:

Some students are scholarship holders	Given True
= Some students are not non-scholarship holders	True by obversion

Since we have got the required subject (student) and the required predicate (non-scholarship holders), we will not use any more eduction. Instead, we will apply "Square of Opposition". If "Some students are not non-scholarship holders" is true, then "All students are non-scholarship holders" is *False* by contradictory relation.

(ii) If "All logicians are mathematicians" is true what can be said about "Some non-mathematicians are not non-logicians"?

Solution:

All logicians are mathematicians	Given True
= All non-mathematicians are non-logicians	True by contraposition
Therefore, some non-mathematicians are not non-logicians	False by contradictory relation

(iii) If "No philosophers are businessmen" is true, what can be said about "No businessmen are non-philosophers"?

Solution:

No philosophers are businessmen	Given True
= No businessmen are philosophers	True by conversion
= All businessmen are non-philosophers	True by obversion
Therefore, no businessmen are non-philosophers	False by contrary relation

(iv) If "No cats are carnivorous" is false, what can be said about "All non-cats are non-carnivorous"?

Solution :

No cats are carnivorous animals	Given False
= No carnivorous animals are cats	False by conversion
= Some non-cats are not non-carnivorous animals	False by contraposition

Therefore, "All non-cats are non-carnivores animals" is True by Contradictory relation.

(v) If "Some men are happy beings" is true what can be said about "All happy beings are men"?

Solution:

Some men are happy beings	Given True
Some happy beings are men	True by conversion

Therefore, "All happy beings are men" is undetermined by superaltern relation.

There is another method to solve problems of this kind. Consider the following example: If "All men are mortal" is true what may be inferred about the truth and falsehood of the following propositions:

1. No men are non-mortal.
2. Some non-mortals are not non-men.
3. All mortals are men.
4. No non-men are non-mortals
5. Some non-mortals are men.
6. No mortals are non-men.
7. No non-mortals are men.
8. All non-men are non-mortals.

9. All non-mortals are non-men.
10. Some men are not mortals.

To infer values of the above ten propositions, first make a list of all possible immediate inferences of the given proposition "All men are mortal". This can be done by converse, obverse and contraposition. All inferred propositions will be true, for all immediate inferences are equivalent propositions.

List of possible immediate inferences of the given proposition "All men are mortal" are as :

(i) Some mortals are men (converse of the given proposition and it is true)
(ii) Some mortals are not non-men (obverse of (i) and is true)
(iii) No men are non-mortals (obverse of the given proposition and is true)
(iv) No non-mortals are men (converse of (iii) and is true)
(v) Some non-men are not mortals (contraposition of (iv) and is true)
(vi) All non-mortals are non-men (contraposition of the given proposition and is true)
(vii) Some non-men are non-mortals (converse of (vi) and is true).

Now, infer the values of the required propositions one by one.

Solutions:

1. The value of "No men are non-mortal" is *true* by (iii) (obverse of the given proposition).
2. The value of "Some non-mortals are not non-men" is as follows:
   All non-mortals are non-men is true by (vi).
   ∴ Some non-mortals are not non-men is *false* by Contradictory relation.

3. The value of "All mortals are men" is as follows:
   Some mortals are men is true by (i).
   ∴ All mortals are men is *Undetermined* by Superaltern.
4. The value of "No non-men are non-mortals" is as follows:
   Some non-men are non-mortals is true by (vii).
   ∴ No non-men are non-mortal is *false* by Contradictory relation.
5. The value of "Some non-mortals are men" is as follows :
   No non-mortals are men is true by (iv).
   ∴ Some non-mortals are men is *false* by Contradictory relation.
6. The value of "No mortals are non-men" is as follows:
   Some mortals are not non-men is true by (ii)
   ∴ No mortals are non-men is *Undetermined* by Superaltern.
7. The value of "No non-mortals are men" is *true* by (iv)
8. The value of "All non-men are non-mortals" is as follows:
   Some non-men are non-mortals is true by (vii)
   ∴ All non-men are non-mortals is *Undetermined* by Superaltern.
9. "All non-mortals are non-men" is *true* by (vi).
10. The value of "Some men are not mortals" is *false* because it is Contradictory of the given proposition "All men are mortals".

Immediate Inference (Eduction) in Modern Logic

The traditional "Square of Opposition" and other forms of immediate inference, say conversion, obversion and contraposition have been criticized by the modern logicians in the context of existential import. In the chapter 3B of this book you have already read that A and E propositions are existentially negative whereas particular propositions I and O are existentially affirmative. If this standpoint is accepted, then conversion of A into I and contraposition of E into O will become invalid. The

conclusion of I from A premiss and O from E premiss is against the basic tenets of deductive logic. In deductive reasoning, the conclusion cannot have anything extra which the premiss or premisses do not have. It will be to "go beyond the evidence". An existentially empty proposition cannot imply existentially loaded proposition. I or O proposition (conclusion) have existential import, whereas A or E (premiss) does not have existential import. Therefore, from A premiss, I conclusion cannot be validly inferred. Similarly, from E premiss, O conclusion cannot be validly inferred. Eduction thus in the Modern Logic takes the following form:

Conversion:	E into E
	I into I

Conversion of A and O propositions are not valid.

Obversion	A into E
	E into A
	I into O
	O into I
Contraposition	A into A
	O into O

There is no valid contraposition of E and I propositions in Modern Logic.

A question may arise : Is there any way to save traditional logic and all the forms of immediate inferences (eduction as well as "Square of Opposition")? This is possible provided we allow that all the propositions universals and particulars have existential import. But if all the categorical propositions, universal and particular, have existential import, then the relationship of contradiction will become invalid and if that happens then the entire logic will collapse. The contradictory relation states that two contradictory propositions, theorems viewpoints, or standpoints cannot be accepted true at the same time. Similarly they cannot be false together at the same time. But if all the categorical propositions have existential import, then A and O

propositions (which are contradictory) can both be false at the same time. For instance, the proposition "All golden mountains of the world are situated in India" and "Some golden mountains are not situated in India" are both false propositions because the class of "golden mountain" is empty. Thus, if all the categorical propositions have existential import, then we will have to pay heavier price in logic. In order to avoid all that, the modern logicians have assigned existential import only to the particular propositions, and not to universal.

Exercise 4

1. Give converse, obverse and contrapositive of the following propositions (wherever possible):
    1. No circle is polygon.
    2. Some cats are black.
    3. Some socialists are not democrats.
    4. All fish are vertebrate.
    5. No man is angel.
    6. Some teachers are good writers.
    7. Some traders are not cheaters.
    8. No sinners are saints.
    9. Some good football players are Europeans.
    10. All businessmen are wealthy persons.
    11. Some poets are not idealists.
    12. Some leaders are good statesmen.
    13. All trees are useful things.
    14. No mathematicians are dishonest beings.
    15. All triangles are three-sided polygons.
    16. No giraffes are short necked.
    17. All giraffes are long necked animals.
    18. Some non-artists are non-honest beings.

19. No non-citizens are voters
20. Some non-mammals are not birds.
21. All non-honest persons are non-artists.
22. No non-artists are honest.
23. All scientists are philosophers.
24. Some non-honest persons are artists.
25. Some non-graduates are non-girls.

2. If 'Some students are radicals' is true, what can be inferred about the truth or falsity of the following:
   (a) Some students are not non-radicals.
   (b) All non-radicals are non-students.
   (c) No radicals are students.
   (d) No students are radicals.
3. If 'No politicians are honest' is true, what can be inferred about the truth or falsity of the following:
   (a) Some non-honests are not non-politicians.
   (b) Some honest persons are not non-politicians.
   (c) No non-politicians are honest.
   (d) Some non-politicians are not non-honest.
4. If the proposition "No soldiers are officers" is true, what can be inferred about each of the following? Give reasons:
   (a) Some soldiers are non-officers.
   (b) Some officers are soldiers.
   (c) Some non-soldiers are non-officers.
   (d) All non-officers are non-soldiers.
5. Assume that "All journalists are pessimists" is true, what can be inferred about the truth or falsity of the following:
   (a) All journalists are non-pessimists.
   (b) No non-jounalists are non-pessimists.

   (c) Some non-pessimists are non-journalists.
   (d) Some journalists are not non-pessimists.
6. If "Some students are not scholarship holders" is true, what can be inferred about the truth or falsity of the following propositions:
   (a) No scholarship holders are students.
   (b) Some students are scholarship holders.
   (c) Some non-scholarship holders are not non-students.
   (d) No non-students are non-scholarship holders.
7. If "Some students are brave" and "Some students are not brave" are both true, what can be inferred about the truth or falsity of the following:
   (a) All students are brave.
   (b) Some brave persons are not students.
   (c) No non-brave persons are non-students.
   (d) Some students are not non-brave.

Chapter 7 – Section A

Categorical Syllogism

SYLLOGISM is mediate inference having two and only two premisses. The premisses jointly imply the conclusion. Aristole, founder of syllogistic reasoning, recognized that all two premissed reasoning is syllogistic. Syllogisms are classified as categorical or non-categorical. In a categorical syllogism all propositions (two premisses and conclusion) are categorical propositions, that is, A, E, I and O.

All men are mortal.
All professors are men.
Therefore, all professors are mortal.

is an example of categorical syllogism.

In a categorical syllogism there are three and only three propositions (two premisses and a conclusion) and there are three and only three terms. For example, in the syllogism.

All M is P
All S is M
Therefore, all S is P.

S, P, M are terms. **The term common between the premisses is called middle term; subject of the conclusion is minor term; predicate of the conclusion is major term.** Each term in a categorical syllogism occurs twice. In the above example,

S	-	Minor term
P	-	Major term
M	-	Middle term

There are two premisses in a categorical syllogism, major premiss and minor premiss. **The premiss which contains major term is called major premiss and the premiss which**

contains minor term is called minor premiss. See the following example :

All M is P	Major premiss
All S is M	Minor premiss
Therefore, all S is P	Conclusion

In a standard form categorical syllogism, major premiss comes first, then the minor premiss occurs and the conclusion comes in the end. A syllogism may not be in a standard form always. The premisses and the conclusion may not be in the right order. The conclusion may sometimes be stated first and premisses later on. In such cases the syllogism is to be rearranged in the proper order and then the validity of the syllogism is determined.

The main characteristic of a categorical syllogism is that all the three propositions (two premisses and conclusion), are categorical propositions A, E, I or O. Singular propositions like "Socrates is mortal". "Socrates is a man" in a syllogism like,

All men are mortal
Socrates is a man.
Therefore, Socrates is mortal.

are treated as universal affirmative A proposition.

Figures of Syllogism

Position of the middle term in the premisses decides the figure of a syllogism. The middle term can occur in four different ways in a standard form categorical syllogism.

First Figure,	M P
	S M
Therefore,	S P

In the first figure, middle term is subject in the major premiss and predicate in the minor premiss.

Second Figure,	P M
	S M
Therefore,	S P

Middle term is predicate in both the premisses in the second figure.

Third Figure,	M P
	M S
Therefore,	S P

Middle term is subject in both the premisses in the third figure.

Fourth Figure,	P M
	M S
Therefore,	S P

Middle term is predicate in the major premiss and subject in the minor premiss in the fourth figure.

Moods of Syllogism

Mood of a syllogism means arrangement of premisses and conclusion. For instance AAA mood in the first figure is expressed as:

All M is P.
All S is M.
Therefore, all S is P.

In AAA mood, the first letter (A) stands for the major premiss, second letter (A) stands for the minor premiss and the last letter (A) stands for the conclusion.

AEE mood in the second figure is expressed as :

All P is M.
No S is M.
Therefore, no S is P.

The first letter (A) is for the major premiss, second letter (E) is for the minor premiss and the last letter (E) is for the conclusion.

Standard Form Categorical Syllogism

The validity of a syllogism can be determined only when the syllogism is in the standard form because rules of validity of a syllogism are applicable only to the standard form syllogism. In the standard form categorical syllogism major premiss must occur

first and the minor premiss comes after that, and the conclusion is in the end. But sometimes conclusion may not be in the right place; it may come either in the beginning or between the premisses. In that case the syllogism has to be rearranged in the proper form. **In order to rearrange the non-standard form categorical syllogism into standard form first the conclusion should be located.** Afterwards determine the minor and the major terms. The subject of the conclusion is minor term and the predicate of the conclusion is major term. Thereafter major premiss and minor premiss can be located easily, for the major premiss contains major term and the minor premiss contains minor term.

There are words which are conclusion indicators and there are words which are premiss indicators. Some of the conclusion indicator words are :

so, hence, it follows, subsequently, it is implied, therefore, thus, etc. Some of the premiss indicators words are :

since, for, because, reason being, on account of, etc.

Let us examine a few syllogisms which are not in standard form and then see how they are reduced into the standard form categorical syllogism.

(i) Some dinosaurs were carnivorous but all dinosaurs are extinct, hence some carnivorous animals are extinct.

The standard form categorical syllogism is:

All dinosaurs are extinct.
Some dinosaurs were carnivorous animals.
Therefore, some carnivorous animals are extinct.

This is AII mood in the third figure.

(ii) No one who ever dies are phoenix and all phoenix are immortals, so no one whoever dies is immortal.

The standard form categorical syllogism is:

All phoenix are immortals.
No one who ever dies are phoenix.
Therefore, no one whoever dies is immortal.

This is **AEE** mood in the first figure.

After rearranging the premisses and the conclusion in a proper order, examine all the propositions (premisses as well as conclusion), whether each one of them is in the standard logical form or not. All the propositions should be in one of the following forms:

All S is P.
No S is P.
Some S is P.
Some S is not P.

If either of the premisses or the conclusion is not in the logical form, then it should be reduced into standard logical form proposition. The devices and techniques for doing this is already worked out in the third chapter.

Exercise 5

Identify major term, minor term and middle term of the following syllogisms:

1. All planets move around sun.
   Earth is a planet.
   Therefore, earth moves around sun.
2. No soldier is coward.
   Ram is a soldier.
   Therefore, Ram is not coward.
3. All parrots are birds.
   All crows are birds.
   Therefore, all crows are parrots.
4. No fish is mammal.
   All cats are mammals.
   Therefore, no cats are fish.

5. Some employees are voters.
   No child is a voter.
   Therefore, no child is employee.
6. Some dinosaurs were carnivorous.
   All dinosaurs are extinct.
   Therefore, some extinct animals were carnivorous.
7. Every shark bites.
   No toothless ever bites.
   Therefore, no toothless is shark.
8. Some students are voters.
   All voters are citizens.
   Therefore, some citizens are students.
9. All cowards are unsuccessful people.
   No patriot is coward.
   Therefore, no patriot is unsuccessful people.
10. All logicians are mathematicians.
    Some mathematicians are musicians.
    Therefore, some musicians are logicians.
11. Some successful people are rich.
    Some businessmen are rich.
    Therefore, some businessmen are successful people.
12. All philosophers are scientists.
    Some philosophers are mathematicians.
    Therefore, some mathematicians are scientists.
13. No bats are feathered.
    All bats can fly.
    Therefore, some animals which can fly are not feathered.
14. All naturalists are observant.
    Some observant people are interested in animals.
    Therefore, some people who are interested in animals are naturalists.

15. Some scientists are not credulous.
All psychiatrists are credulous.
Therefore, some psychiatrists are not scientists.

Exercise 6

Find out major premiss, minor premiss and determine figures and moods of the following syllogisms:

Note: If a syllogism is not in the standard form then first change it into standard form categorical syllogism, and then proceed further.

(1) All saints are polite. No sinners are saints. Therefore, no sinners are polite.

(2) Some professors are not good orators. All good orators are successful persons. Thus, some professors are not successful persons.

(3) All citizens are voters but no child is a voter, so, no child is a citizen.

(4) No one who believes in human equality are fascist and all those who believe in human equality are strong defenders of democracy, subsequently no strong defenders of democracy are fascists.

(5) Some trade unions are politically-motivated organizations. But some trade unions are for the welfare of the workers. From this, one concludes that some politically-motivated organizations are for the welfare of workers.

(6) No coward is self-confident person because all self-confident persons are mentally strong whereas no coward is mentally strong person.

(7) No insects are eight-legged animals. A wasp is a insect. Hence, a wasp is not eight-legged animal.

(8) No Indian woman is a soldier. All soliders serve the country. Therefore, some who serve their country are not Indian women.

(9) No civilized person is rude in his behaviour, for all civilized persons are considerate of others and no one who is considerate of others is rude in his behaviour.

(10) All enterprising people have dynamic personality since all those who have dynamic personality look forward and all those who look forward are enterprising people.

Chapter 7 – Section B

Validity of Categorical Syllogism: Traditional Method

ARISTOTLE and other traditional logicians devised certain rules which determine the validity of a syllogism. These rules are mainly based on the distribution of terms. A valid syllogism has to obey all the given rules, and if any of the rules is violated, then the syllogism becomes invalid.

Rule No. 1

A syllogistic reasoning must have three and only three terms. Since each term occurs twice, the term should be used in the same sense in both of its occurrences. The meaning and context of a term at both places of its occurrences should be same. Sometimes a term carries particular meaning at one place and is used in different sense at another place. In that case the syllogism is invalid and has the **fallacy of four terms.** For example:

All laws are made by governments

v = at is a law of falling bodies

Therefore, governments made v = at.[1]

In this syllogism the term "law" is ambiguous. It refers to a physical law (such as the law of falling bodies) and also to a legislative law. As a result, this syllogism has four terms instead of three, and hence is invalid having fallacy of four terms.

Sometimes more than three terms are used but two of them are either synonymous or contradictory. In that case, terms of the syllogism can be reduced to three with the help of conversion, obversion or contraposition. For example:

1. *The World Book Encyclopedia,* vol. 12, p. 362.

All librarians are faculty members.

Some staff members are non-faculty members.

Therefore, some staff members are not librarians.

In this syllogism, there are initially four terms namely:

1. librarians
2. faculty members
3. non-faculty members
4. staff members

But the term non-faculty member can be replaced by faculty member after obverting minor premiss. The minor premiss then becomes

Some staff members are not faculty members.

The syllogism now has only three terms and it is stated as:

All librarians are faculty members.

Some staff members are not faculty members.

Therefore, some staff members are not librarians.

In case of synonymous terms, one synonymous term is replaced by another synonymous term because meaning of the synonymous terms is same. However, if the terms are more than three and none of them is synonymous or contradictory, then there is fallacy of four terms.

Rules related to distribution of terms

Rule No. 2

Middle term must be distributed at least at one of the places of its occurrences. If the middle term is not distributed in any of the premisses, then the syllogism becomes invalid and has the **fallacy of undistributed middle.** For instance, in the following syllogisms, the middle term is not distributed in any of the premisses and hence, they are invalid syllogisms.

(i) All P is M.
All S is M.
Therefore, all S is P.

(ii) All cows are mammals.
All cats are mammals.
Therefore, all cats are cows.

Rule No. 3

A term which is distributed in the conclusion must necessarily be distributed in the premiss. We had discussed it earlier also in the immediate inference. A syllogism becomes invalid if a term is distributed in the conclusion but undistributed in the premiss. A categorical syllogism having minor term distributed in the conclusion but undistributed in the minor premiss, is invalid having the **fallacy of illicit minor.** For example:

(i) All M is P.
All M is S.
Therefore, all S is P.

The term "S" is distributed in the conclusion but not in the premiss. So it is invalid syllogism having fallacy of illicit minor.

(ii) No mathematicians are musicians.
All mathematicians are scientists (undistributed).
Therefore, no scientists (distributed) are musicians.

In this syllogism there is fallacy of illicit minor and hence it is invalid syllogism.

Similarly, the categorical syllogism having major term distributed in the conclusion but undistributed in the major premiss is invalid, having the **fallacy of illicit major**. For example :

(i) All M is P.
Some S is not M.
Therefore, some S is not P.

(ii) All mathematicians are logicians (undistributed).
Some businessmen are not mathematicians.
Therefore, some businessmen are not logicians (distributed)

are invalid syllogisms having fallacy of illicit major.

To commit either the fallacy of illicit major or illicit minor means the syllogism violates the very basic tenet of the deductive logic. In the deductive logic the conclusion is already contained in the premisses, that is why, the conclusion cannot be more comprehensive than the premisses. If a term is distributed in the conclusion but not in the premiss, then it means the conclusion contains more than the premisses which is not permitted in the deductive reasoning.

Rules of Quality

There are two rules governing the qualities of premisses and conclusion. They are as follows :

Rule No. 4

A valid syllogism must not have two negative premisses. A syllogism having both the negative premisses is invalid and has the **Fallacy of Exclusive Premisses.**

The reasoning behind this rule is that two negative propositions have nothing common. There is no connective thread between the premisses.

The function of middle term M is to relate S and P, but here M does not connect them.

(i) No P is M.
No S is M.
Therefore, no S is P.

is invalid syllogism having the fallacy of exclusive premisses.

Similarly, the following syllogism is invalid having the same fallacy.

(ii) No cats are flying creatures.
Some domestic animals are not cats.
Therefore, some domestic animals are not flying creatures.

Rule No. 5

If one of the premisses is negative and the conclusion is affirmative then it makes invalid syllogism. It has the **Fallacy of drawing**

affirmative conclusion from a negative premiss[2]. For example :

(i) All P is M.
No S is M.
Therefore, all S is P

is invalid having the fallacy of drawing affirmative conclusion from a negative premiss. The following syllogism also has the same fallacy.

All athletes are strong-built persons.
No strong-built person is lazyman.
Therefore, all lazymen are athletes.

Rule of Quantity

Rule No. 6

A syllogism having both the universal premisses must have universal conclusion. A syllogism having both universal premisses and a particular conclusion is invalid having the fallacy called **Existential Fallacy.** For example.

All M is P.
No S is M.
Therefore, some S is not P.

is invalid syllogism because of existential fallacy. However, this rule was not there in the traditional logic. For the traditional logicians syllogisms like:

(i) All M is P.
All S is M.
Therefore, some S is P.

(ii) No M is P.
All M is S.
Therefore, some S is not P.

2. The name of fallacy is from Irving M. Copi., and Carl Cohen, *Introduction to Logic* (ninth edition), p. 265.

(iii) All P is M.
All M is S.
Therefore, some S is P.

(iv) No P is M.
All M is S.
Therefore, some S is not P.

are valid. But for the modern logicians all these syllogistic moods are invalid for they commit existential fallacy.

We have already discussed existential import in the third chapter, and stated that A and E do not have existential import whereas I and O propositions have it. In deductive logic conclusion cannot possess anything which the premisses do not have. The conclusion cannot be particular if all the premisses are universal.

Corollary 1: From two particular premisses no valid mood follows in a standard form categorical syllogism.

Proof : If both the premisses are particular, the combinations of premisses are : II, IO, OI and OO.

II combination of premisses do not distribute any term and hence the syllogism having II as premisses is always invalid having the fallacy of undistributed middle. The combination of OO premisses always makes invalid syllogism because of the fallacy of exclusive premisses.

We are left with IO and OI combinations of premisses. From IO combination of the premisses four possible moods can be constructed which are as : IOA, IOE, IOI and IOO.

Out of these moods IOE and IOO are invalid because of illicit major (though there is one more fallacy also in them depending on the figure to which the syllogism belongs).

IOA and IOI are invalid moods having the fallacy of drawing affirmative conclusion from a negative premiss.

Similarly, from OI combinations of premisses we get four syllogistic moods. OIA, OIE, OII and OIO. OIA and OII are

invalid for they have the fallacy of drawing an affirmative conclusion from a negative premiss. OIE definitely has the fallacy of illicit minor though it has one more fallacy also OIO has either fallacy of illicit major or undistributed middle.

Hence, from two particular premisses no valid categorical syllogism can be constructed.

Corollary 2 : If one of the premisses is particular, then the conclusion has to be particular in a valid standard form categorical syllogism.

or

From one particular premiss, universal conclusion cannot be drawn in a valid standard form categorical syllogism.

Proof : The possible combinations of the premisses with one particular premiss are : AI, IA, AO, OA, EI, IE, EO and OE.

Out of these EO and OE combinations of premisses will always make invalid syllogism because of the fallacy of exclusive premisses.

Let us consider all other combinations of premisses one by one. The moods with AI combination of premisses and universal conclusion are : AIA and AIE.

Both of them are invalid syllogisms because of illicit minor, although they have one more fallacy also depending on the figure to which the mood belongs.

From IA again we get two moods : IAA and IAE.

Whereas, IAE definitely has the fallacy of illicit major, IAA has either the fallacy of illicit minor or the fallacy of undistributed middle.

From AO premisses, the two required moods are: AOA and AOE.

AOA is invalid for it has the fallacy of drawing an affirmative conclusion from a negative premiss. AOE has either the fallacy of illicit major or illicit minor depending on the figure to which the mood belongs.

Similarly, from OA we get two moods: OAA and OAE.

OAA has the fallacy of drawing an affirmative conclusion from a negative premiss, though it has one more fallacy also. OAE has either the fallacy of illicit major or illicit minor depending on the figure to which the mood belongs.

From EI premisses we get EIA and EIE moods. They both have the fallacy of illicit minor.

IE combination of premisses give us IEA and IEE moods.

IEA is invalid mood for it has fallacy of drawing an affirmative conclusion from a negative premiss. IEE has the fallacy of illicit major.

Thus, from a particular premiss, universal conclusion cannot be drawn in a valid standard form categorical syllogism.

Corollary 3 : In a valid standard form of categorical syllogism no negative conclusion follows from two affirmative premisses.

Proof: Combinations of both affirmative premisses are AA, AI, IA and II.

The combination of II premisses always make invalid syllogism because it does not distribute any term in the premiss and, hence, has the fallacy of undistributed middle.

AA premisses with a negative conclusion form AAO and AAE, moods.

AAO suffers from the existential fallacy and AAE has either the fallacy of illicit major or undistributed middle depending on the figure of the mood.

AI and IA combinations of premisses and a negative conclusion gives us AIO, AIE, IAO and IAE, moods.

Whereas, IAO and IAE definitely have the fallacy of illicit major, AIE has the fallacy of illicit minor. AIO has either the fallacy of illicit major or undistributed middle depending on the figure.

The application of these two rules gives the following valid moods in the first figure.

AAA	Barbara
EAE	Celarent
AII	Darii
EIO	Ferio

The names of these valid moods were given by the medieval thinkers. The names contain the vowel symbols. The order in which the vowels occur in the name is the same as that of the mood. The 1st vowel stands for the major premiss, 2nd for the minor premiss and the last for the conclusion. For instance, Barbara, represents AAA, Celarent is EAE and so on. The logicians provided the names of valid moods in order to memorize them easily.

Special rules of 2nd figure

1. One of the premisses must be negative. (The violation of the rule leads to the fallacy of undistributed middle.)
2. The major premiss must be universal (otherwise there is fallacy of illicit major).

Proof of the 1st rule: Format of 2nd figure is :

PM
SM

Therefore, S P.

One of the premisses must be negative. If both the premisses are affirmative then the middle term is not distributed in any of the premisses. In both the premisses the middle term is predicate, and predicate term is not distributed in affirmative propositions. So if both the premisses are affirmative in second figure, then there is fallacy of undistributed middle. Hence one of the premisses must be negative in the second figure.

Proof of the 2nd rule: The major premiss must be universal in the 2nd figure. Since middle term is predicate in both the premisses, one of the premisses has to be negative in order to make middle term distributed. One negative premiss means the conclusion is negative. The negative conclusion distributes its predicate term which is a major term. In a valid categorical syllogism, the major term if distributed in the conclusion then it must necessarily be distributed in the major premiss. But this is possible only if major premiss is universal, for only in a universal proposition A or E, subject term is distributed. If major premiss is not universal in second figure then the syllogism becomes invalid having the fallacy of illicit major. Hence, major premiss must be universal in the second figure.

Valid moods of the second figure are :

AEE	Camestres
EAE	Cesare
AOO	Baroco
EIO	Festino

Special rules of 3rd figure

1. The conclusion must be particular (otherwise there is fallacy of illicit minor).
2. The minor premiss must be affirmative (otherwise there is fallacy of illicit major).

Proof of the 1st rule: Format of 3rd figure is :

MP
MS

Therefore, S P

The conclusion must be particular in the third figure. If the conclusion in the third figure is universal then minor term S is distributed in the conclusion. In a valid syllogism, if a term is distributed in the conclusion, it must be distributed in the premiss also. In order that the minor term is distributed in the minor premiss, the premiss must be negative, for only in a negative proposition the predicate term is distributed. The negative minor premiss implies a negative conclusion. In turn negative conclusion makes its predicate, which is major term, distributed. Now major

term must be distributed in the major premiss also. Since major term in the major premiss is predicate it can be distributed there only if major premiss is negative. But two negative premisses do not make a syllogism valid. Thus, if conclusion is not particular in the third figure, the syllogism is invalid having the fallacy of illicit minor.

Proof of the 2nd rule: The minor premiss must be affirmative in the third figure. If the minor premiss is not affirmative but instead a negative proposition, then in a valid categorical syllogism it implies two things:

(i) a negative conclusion, and

(ii) affirmative major premiss.

The negative conclusion means its predicate term (major term) is distributed in the conclusion. In a valid syllogism the major term then must necessarily be distributed in the major premiss also. But major premiss being affirmative does not distribute its predicate, for in the third figure major term is predicate. So major term is distributed in the conclusion but undistributed in the premiss which leads to the fallacy of illicit major. Thus, if the minor premiss is not affirmative in the third figure, then the syllogism will become invalid having the fallacy of illicit major.

Valid moods in the third figure are:

AII	Datisi
IAI	Disamis
OAO	Bocardo
EIO	Ferison

Special rules of 4th figure

1. If either premiss is negative, the major premiss cannot be particular (otherwise there is fallacy of illicit major).
2. If the major premiss is affirmative, the minor premiss cannot be particular (otherwise there is fallacy of undistributed middle).

3. If the minor premiss is affirmative, then the conclusion cannot be universal (otherwise there is fallacy of illicit minor).

Proof of 1st rule: Format of fourth figure is :

PM
MS

Therefore, S P.

If either of the premisses is negative, the conclusion is also negative. The negative conclusion distributes its predicate term which is major term. The major term then must necessarily be distributed in the major premiss. But if the major premiss is not universal, then its subject term is not distributed. The syllogism then becomes invalid having the fallacy of illicit major. Thus, if one of the premisses is negative in the fourth figure, then the major premiss must necessarily be universal.

Proof of 2nd rule: If the major premiss is affirmative, then the middle term is not distributed there because in a affirmative proposition, predicate term is undistributed. The middle term then has to be distributed in the minor premiss. This is possible only if the minor premiss is a universal proposition. For the middle term is subject in the minor premiss in the fourth figure, and subject term is distributed only in the universal proposition. Therefore, if the major premiss is affirmative, then the minor premiss cannot be particular otherwise the syllogism becomes invalid having the fallacy of undistributed middle.

Proof of 3rd rule: If the minor premiss is affirmative, the conclusion cannot be universal. The affirmative minor premiss and the universal conclusion makes the syllogism invalid in the fourth figure having the fallacy of illicit minor. The universal conclusion distributes its subject term, because universal proposition (A or E) always distributes its subject term. Now if minor term is distributed in the conclusion then it must be distributed in the minor premiss. But the minor premiss being affirmative does not distribute its predicate term because in the

fourth figure minor term is predicate term. Hence, there is fallacy of illicit minor. So in the fourth figure if minor premiss is affirmative, the conclusion must be particular.

Valid moods in the fourth figure are :

AEE	Cameres
IAI	Dimaris
EIO	Fresison

Its amazing EIO mood is valid in all the four figures, whereas IEO is invalid in all the figures.

256 possible moods can be constructed with different combinations of categorical propositions in all the four figures of syllogism. Out of them only 15 moods are valid in all. In the first figure 4 moods, in the second figure 4 moods, in the third figure 4 moods and in the fourth figure only 3 moods are valid which shows validity is so rare and precious.

Let us see how different combinations of A, E, I and O propositions yield 256 moods.

Taking "A" as major premiss, we get the following moods:

AAA	AIA
AAE	AIE
AAI	AII
AAO	AIO
AEA	AOA
AEE	AOE
AEI	AOI
AEO	AOO

There are 16 combinations and each one of them further has four moods in four different figures.

For instance, AAA combination of premisses and conclusion has four moods, one each in 1st, 2nd, 3rd and 4th figure. So each one of 16 moods has four moods further. Thus if A is major premiss then 64 moods can be framed (how many of them are valid is a separate question).

Similarly, 64 moods can be framed by taking E as major premiss, 64 moods are there taking I as a major premiss, and further 64 moods taking O as a major premiss.

EAA	IAA	OAA
EAE	IAE	OAE
EAI	IAI	OAI
EAO	IAO	OAO
EEA	IEA	OEA
EEE	IEE	OEE
EEI	IEI	OEI
EEO	IEO	OEO
EIA	IIA	OIA
EIE	IIE	OIE
EII	III	OII
EIO	IIO	OIO
EOA	IOA	OOA
EOE	IOE	OOE
EOI	IOI	OOI
EOO	IOO	OOO

Out of 256 moods, only 15 are valid. A student can apply six rules of syllogism to eliminate the invalid ones. AAI and EAO moods in first figure, third figure and fourth figure are valid according to traditional logicians. But in the context of existential import these moods have become invalid having the existential fallacy.

Exercise 7

A. Test the validity/invalidity of the following syllogistic forms by the traditional method of rules and fallacies in the figures indicated against them.

1. AAA - 2	7. AOO - 4	13. IOE - 2
2. AEE - 3	8. EIE - 3	14. EIE - 4
3. AEO - 1	9. IAA - 2	15. IAA - 4
4. AII - 4	10. IOO - 1	16. OIO - 1
5. AOE - 1	11. III - 4	17. OAI - 3
6. AOI - 3	12. IEE - 3	18. OAE - 1

19. OOO - 4	30. OAI - 1	41. AAO - 1
20. IAI - 1	31. OAO - 1	42. OAO - 4
21. EAO - 3	32. OEO - 4	43. EOO - 2
22. EAE - 4	33. EAO - 1	44. EIE - 1
23. AEO - 3	34. AAA - 4	45. IEO - 2
24. EAE - 2	35. AAE - 1	46. IAA - 3
25. AAI - 1	36. OAO - 2	47. AAA - 1
26. OAI - 2	37. EAO - 4	48. AEO - 4
27. IEO - 3	38. IEO - 1	49. AAA - 3
28. OAA - 4	39. EIO - 3	50. EIO - 1
29. EAE - 3	40. EOO - 4	

B. Test the validity/invalidity of the following syllogisms by the traditional method of rules and fallacies, after rearranging them into standard form (wherever necessary):

1. Some extremists are violent persons and no doctors are extremists. It follows no violent persons are doctors.
2. All applicants are degree holders and Shiela is not an applicant. Hence, Shiela is not degree holder.
3. All teachers are critical thinkers and all critical thinkers are good analysers. So it is true all good analysers are teachers.
4. All those who eat spicy foods are ulcer patients because all spicy foods are unhealthy food and all ulcer patients eat unhealthy food.
5. Since all dictators are ruthless and no ruthless is lover of art, so some lover of art are not dictators.

6. Some games are played for entertainment and some games are played for making profit. Thus, some entertainment acts are for making profit.
7. All citizens are voters whereas some citizens are not government employees. Therefore, some government employees are not voters.
8. No Indian is Greek but some Indians are Aryans, therefore, some Greeks are Aryans.
9. Since no slow thinker solves mathematical problems quickly and Sita is not a slow thinker, so Sita solves the mathematical problems quickly.
10. All soldiers are slaves because all slaves are loyal and all soldiers are loyal.

C. Examine the validity/invalidity of the syllogisms given in the Exercise No. 6

In addition to the six rules which are treated as self-evident and self-explanatory, there are certain corollaries (or hypotheses) which are also applicable to all the moods of syllogism. They are called corollaries or hypotheses because they are not self-evident, and can be proved with the help of the above given six rules.

Chapter 7 – Section C

Validity of Categorical Syllogism

Modern Method

THE rules and fallacies for the categorical syllogisms provided by the traditional logicians are primarily based on the distribution of terms. The traditional method of checking validity of syllogism were basically non-mathematical. The modern logicians suggested other alternative methods to determine the validity of the categorical syllogisms. Some of these methods are:

(i) Venn diagram

(ii) Anti-logism Theorem by Mrs. Christine Ladd-Franklin

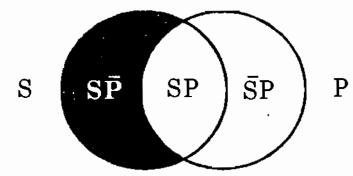
(iii) Method provided in the Predicate Calculus – Quantifiers.

The present chapter, however, deals with the first two methods. Chapter 15 deals with Predicate Calculus and Quantifiers. We start with Venn diagram.

The basis of validity of the categorical syllogisms in the Venn diagram is same as that of the traditional logic, that is, **the conclusion must not contain anything which the premisses do not have.** The logicians examine whether premisses are sufficient or efficient enough to support the conclusion or not. If the conclusion is implied by the premisses jointly then the syllogism is valid, and if the premisses do not imply the conclusion, the syllogism is invalid.

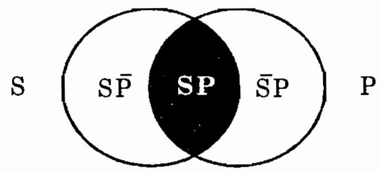
In Venn diagram the terms of a categorical proposition are represented by circles. We have already seen how A, E, I, O propositions are expressed in Venn diagrams in the third chapter. The shaded portion on the diagrams represent empty class. Let us again see how the categorical propositions are shown through diagrams:

All S is P



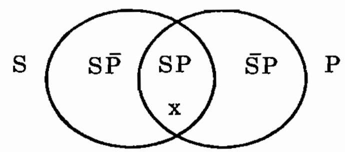
$S\bar{P} = 0$

No S is P



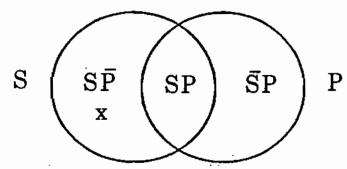
$SP = 0$

Some S is P



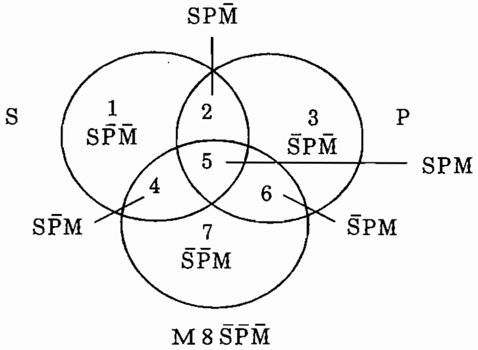
$SP \neq 0$

Some S is not P



$S\bar{P} \neq 0$

In a categorical syllogism there are three terms. In a diagram representing categorical syllogism there are three circles overlapping each other as has been shown in the following example:



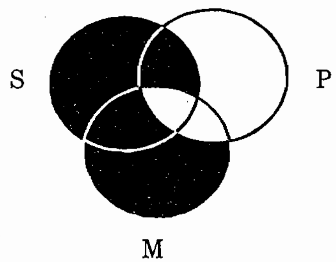
Look at the following illustrations of various categorical syllogistic moods:

We begin with AAA mood in 1st figure:

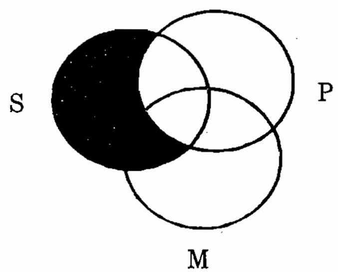
All M is P.
All S is M.

Therefore, all S is P.

Premisses



Conclusion



major premiss is $\bar{S}\bar{P}M + S\bar{P}M = 0\}$

minor premiss is $S\bar{P}\bar{M} + SP\bar{M} = 0\}$

conclusion is $S\bar{P}\bar{M} + S\bar{P}M = 0$

Since the sum of two segments is zero, then each segment separately is also zero.

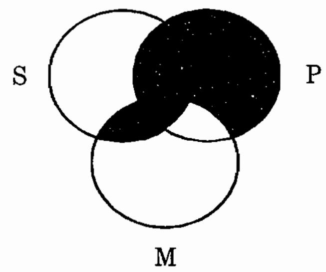
major premiss	$\bar{S}\,\bar{P}\,M = 0$
	$S\,\bar{P}\,M = 0$
minor premiss	$S\,\bar{P}\,\bar{M} = 0$
	$S\,P\,\bar{M} = 0$
conclusion	$S\,\bar{P}\,\bar{M} = 0$
	$S\,\bar{P}\,M = 0$

The premisses jointly imply the conclusion. The diagram also clearly shows the conclusion is endorsed by both the premisses together. Hence, it is a valid syllogistic form.

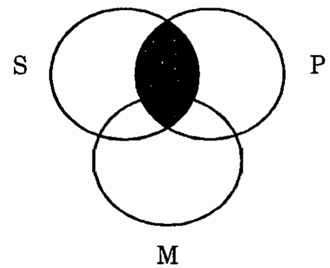
2. AEE in 2nd figure:

All P is M.
No S is M.
Therefore, no S is P.

Premisses



Conclusion



The diagram clearly shows that conclusion is covered by the premisses.

major premiss	$\bar{S}\,P\,\bar{M} + S\,P\,\bar{M} = 0$
	$\bar{S}\,P\,\bar{M} = 0$
	$S\,P\,\bar{M} = 0$
minor premiss	$S\,P\,M + S\,\bar{P}\,M = 0$
	$S\,P\,M = 0$
	$S\,\bar{P}\,M = 0$
conclusion	$S\,P\,M + S\,P\,\bar{M} = 0$

The premisses together state:

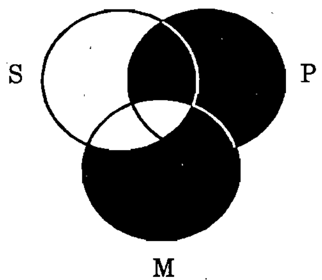
$$\bar{S}\,P\,\bar{M} = 0$$
$$S\,P\,\bar{M} = 0$$
$$S\,P\,M = 0$$
$$S\,\bar{P}\,M = 0$$

and they jointly imply the conclusion. Therefore, the syllogistic mood is valid.

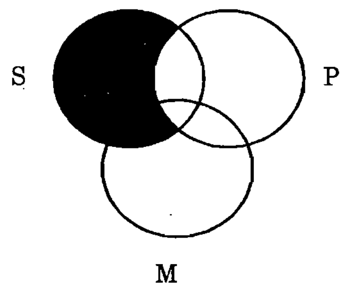
3. AAA in 4th figure:

   All P is M.
   All M is S.
   Therefore, all S is P.

Premisses



Conclusion



The diagram shows the conclusion is not covered by the premisses. Therefore, the mood is invalid.

major premiss $S\,P\,\bar{M} + \bar{S}\,P\,\bar{M} = 0$

minor premiss $\bar{S}\,P\,M + \bar{S}\,\bar{P}\,M = 0$

conclusion $S\,\bar{P}\,\bar{M} + S\,\bar{P}\,M = 0$

Since the sum of two segments is zero, each segment individually is zero.

The major premiss states $S\,P\,\bar{M} = 0$

$\bar{S}\,P\,\bar{M} = 0$

The minor premiss states $\bar{S}\,P\,M = 0$

$\bar{S}\,\bar{P}\,M = 0$

Jointly they state:

$$S\,P\,\bar{M} = 0$$
$$\bar{S}\,P\,\bar{M} = 0$$
$$\bar{S}\,P\,M = 0$$
$$\bar{S}\,\bar{P}\,M = 0$$

Conclusion $S\bar{P}\,\bar{M} = 0$

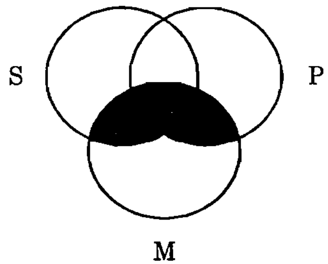
$$S\,\bar{P}\,M = 0$$

Notice, no segment of the conclusion is covered by the premisses. Therefore, the mood is invalid.

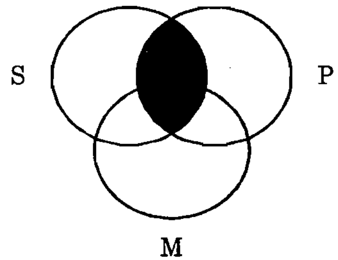
4. EEE mood in 1st figure:

   No M is P.
   No S is M.
   Therefore, no S is P.

Premisses



Conclusion



The conclusion is not completely endorsed by the premisses. Therefore, it is invalid syllogistic form?

major premiss $S\,P\,M + \bar{S}\,P\,M = 0$

minor premiss $S\,P\,M + S\,\bar{P}\,M = 0$

conclusion $S\,P\,M + S\,P\,\bar{M} = 0$

Since the sum of two segments is zero, they separately are also zero, major premiss states:

$S\,P\,M = 0$

$\bar{S}\,P\,M = 0$

minor premiss states:

$S\,P\,M = 0$

$S\,\bar{P}\,M = 0$

conclusion states:

$S\,P\,M = 0$

$S\,P\,\bar{M} = 0$

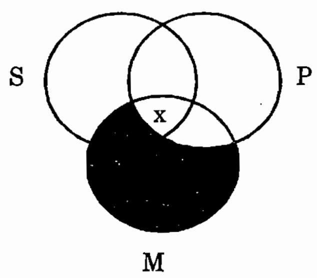
Notice SPM segment of the conclusion which states SPM = 0 is covered by the premisses but none of the premisses state anything about $SP\bar{M}$. Since the premisses are silent about $SP\bar{M}$ and do not imply $SP\bar{M} = 0$, therefore it is invalid mood.

So far we were dealing with the universal (propositions) premisses as well as conclusion which are existentially negative, and in the diagram they represent empty classes.

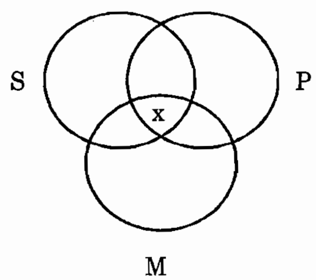
Let us now consider those moods where one of the premisses is particular. For example All in 1st figure :

5. All M is P.
   Some S is M.
   Therefore, some S is P.

Premisses



Conclusion



The diagram shows the conclusion is supported by the premisses.

major premiss states: $\bar{S}\,\bar{P}\,M + S\,\bar{P}\,M = 0$

This means, $\bar{S}\,\bar{P}\,M = 0$

$S\,\bar{P}\,M = 0$

minor premiss states: $S\,\bar{P}\,M + S\,P\,M \neq 0$

From major premiss, we get $S\,\bar{P}\,M = 0$

Therefore, $S\,P\,M \neq 0$. It means SPM definitely has member say x.

The conclusion states: $S\,P\,M + S\,P\,\bar{M} \neq 0$

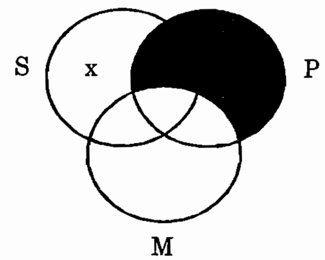
Even if the premisses are silent about $SP\bar{M}$ segment, but they (premisses) jointly and categorically state that SPM has members. Thus the conclusion is upheld and supported by the premisses. The conclusion, after substituting the value of S P M, looks like.

$x + ? \neq o$

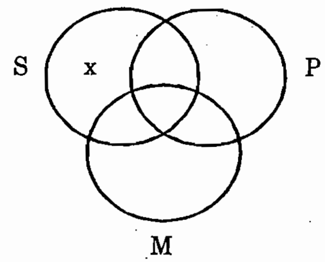
which is correct. So the mood All in 1st figure is valid.

6. AOO mood in 2nd figure:
   All P is M.
   Some S is not M.
   Therefore, some S is not P .

Premisses



Conclusion



major premiss states $\bar{S}\,P\,\bar{M} + S\,P\,\bar{M} = 0$

It implies $\bar{S}\,P\,\bar{M} = 0$

$S\,P\,\bar{M} = 0$

minor premiss states $S\,P\,\bar{M} + S\,\bar{P}\,\bar{M} \neq 0$

Substituting $S\,P\,\bar{M} = 0$ from the major premiss in the minor premiss, we get: $S\,\bar{P}\,\bar{M} \neq 0$, which means $S\,\bar{P}\,\bar{M}$ has members say x.

The conclusion states: $S\bar{P}\bar{M} + S\bar{P}M \neq 0$

Whereas the premisses do not cover the portion $S\bar{P}M$, but they state categorically that the other segment $S\bar{P}\bar{M}$ has members. The premisses jointly endorse the conclusion. Even if $S\bar{P}M$ is not covered by any of the premisses yet,

$x + ? \neq 0$

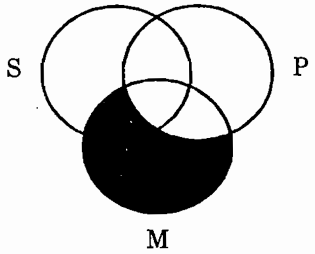
holds good. Therefore, the mood AOO in 2nd figure is valid.

7. Let us examine AOO in 1st figure:

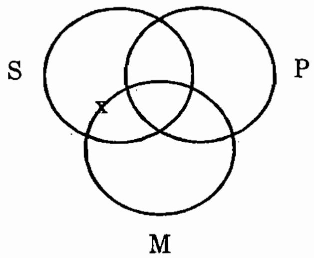
All M is P.
Some S is not M.

Therefore, some S is not P.

Premisses



Conclusion



major premiss states $\bar{S}\bar{P}M + S\bar{P}M = 0$

It implies $\bar{S}\bar{P}M = 0$

$S\bar{P}M = 0$

minor premiss states $S\bar{P}\bar{M} + SP\bar{M} \neq 0.$

But the minor premiss does not very definitely state where exactly the member x lies, that is, whether the members are in $SP\bar{M}$ segment or in $S\bar{P}\bar{M}$ segment or in both $S\bar{P}\bar{M}$ and $SP\bar{M}$.

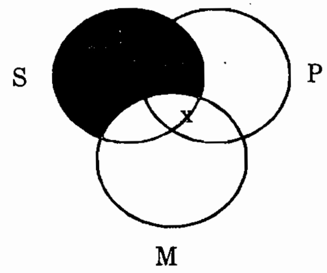
The conclusion states $S\bar{P}\bar{M} + S\bar{P}M \neq 0$. The major premiss states $S\bar{P}M = 0$. Therefore, the minor premiss must categorically state $S\bar{P}\bar{M} \neq 0$. But it does not do that. The premisses are not clear about $S\bar{P}\bar{M}$. Therefore, AOO in the 1st figure is invalid mood.

8. IAI in 2nd figure:

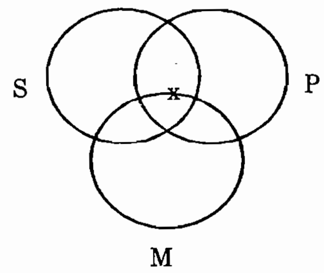
Some P is M.
All S is M.

Therefore, some S is P.

Premisses



Conclusion



major premiss states $SPM + \bar{S}PM \neq 0$

But in which segment the members lie, the premiss does not state. So we have kept x on the border of the intersecting circles,

minor premiss states $S\bar{P}\bar{M} + SP\bar{M} = 0$

which means $S\bar{P}\bar{M} = 0$

$SP\bar{M} = 0$

The conclusion states : $SPM + SP\bar{M} \neq 0$

While S P M has been shown as zero by the premisses, but they do not categorically state S P M has members. The major premiss does not specify where exactly the members lie in the

$SPM + \bar{S}PM \neq 0$

So the mood IAI in second figure is invalid.

Exercise 8

Test the validity/invalidity of the following syllogistic moods by Venn diagram:

1.	EAE - 2		9.	AII - 3	
2.	AEE - 4		10.	IAI - 4	
3.	EAE - 1		11.	IIO - 3	
4.	AAA - 2		12.	OAO - 1	
5.	EAA - 1		13.	IEO - 2	
6.	EAO - 3		14.	OIE - 4	
7.	AAI - 4		15.	IAI - 3	
8.	EIO - 1				

The Antilogism

Mrs Christine Ladd-Franklin had devised yet another method, antilogism, for testing the validity of categorical syllogisms. Antilogism or "inconsistent triad" (as it is popularly known) is a device to test the validity of syllogisms without using the traditional rules. The antilogism is closely associated with the indirect method.

From the syllogistic mood (whose validity one wants to determine) an antilogism is framed replacing the conclusion (of the given mood) by its contradictory. The premisses of the mood remain the same in the antilogism. For instance, in order to test the validity of AAA mood in first figure, the antilogism derived from the mood AAA is AAO.

Mood	Antilogism
All M is P.	All M is P.
All S is M.	All S is M.
Therefore, all S is P.	Therefore, some S is not P.
	(*contradictory of the conclusion*)

The contradictory of the conclusion "All S is P" is "Some S is not P" [By Square of Opposition]. The mood AAA is valid if and only if the antilogism framed by the mood AAA, satisfies all the following three conditions:

(1) The antilogism must have two universal propositions. They both may be the premisses or one of them may be the premiss and the other is conclusion. The third proposition (it may be a premiss or a conclusion) should be particular. The univeral propositions are equal to zero. They are, therefore, equations. You know from Boolean algebra universal propositions are existentially negative and are equated to zero. The particular proposition is inequation. It is not equal to zero.

(2) The equations (universal propositions) must have one common term which occurs affirmatively at one place and negatively at another place. This means the common term must occur affirmatively in one equation and negatively in another equation.

(3) Now examine the inequation, that is, the particular proposition. This proposition must contain all the remaining terms of the universal propositions identically as they occur there.

Example 1: Let us test the validity of AAA mood in first figure by the antilogism method. The mood is:

Mood

All M is P	$\longrightarrow$	$M\bar{P} = 0$
All S is M	$\longrightarrow$	$S\bar{M} = 0$
Therefore, all S is P.		Therefore, $S\bar{P} = 0$

After replacing conclusion by its contradictory, we get the following antilogism:

Antilogism

All M is P	$\longrightarrow$	$M\bar{P} = 0$
All S is M	$\longrightarrow$	$S\bar{M} = 0$
Therefore, some S is not P.		Therefore, $S\bar{P} \neq 0$

(contradictory of A proposition is O).

Now we have to see whether this antilogism satisfies *all* the three conditions or not.

(1) There are two universal propositions (equations = 0) and one particular proposition (inequation ≠ 0) in the antilogism. So the first condition is satisfied by the antilogism.

(2) The two equations $(M)\bar{P} = 0$

$$S(\bar{M}) = 0$$

contain one common term which occurs once affirmatively and once negatively, M and $\bar{M}$. Thus, the second condition is also satisfied.

(3) The inequation (particular proposition), $S\bar{P} \neq O$, contains the remaining terms of equations (universal propositions)

$(M)\bar{P} = 0$

$S(\bar{M}) = 0$

that is, S and $\bar{P}$ as they are in the universal propositions. Since the antilogism satisfies all the three conditions, therefore, the syllogistic mood which imply the antilogism, is valid,

Example 2: AOO mood in second figure.

Mood

All P is M.	⟶	$P\bar{M} = 0$
Some S is not M.	⟶	$S\bar{M} \neq 0$
Therefore, some S is not P.		Therefore, $S\bar{P} \neq 0$

After replacing the conclusion by its contradictory, we get the following antilogism:

Antilogism

All P is M.	⟶	$P\bar{M} = 0$
Some S is not M.	⟶	$S\bar{M} \neq 0$
Therefore, All S is P.		Therefore, $S\bar{P} = 0$

(contradictory of the conclusion O is A).

Let us examine whether the antilogism satisfies *all* the conditions or not:

(1) There are two equations $P\bar{M} = 0$ and $S\bar{P} = 0$. There is one inequation $S\bar{M} \neq 0$.

(2) The equations $(P)\bar{M} = 0$

$$S(\bar{P}) = 0$$

contain one common term, which occurs once affirmatively and once negatively (P and $\bar{P}$).

(3) The inequation $S\bar{M} \neq 0$ contains the remaining terms of both the equations,

$$(P)\bar{M} = 0$$

$$S(\bar{P}) = 0$$

S, $\bar{M}$, exactly as they are there in the equations.

Thus, you notice the antilogism satisfies all the conditions. Therefore, the syllogistic mood from where the antilogism is formed, is valid.

Example 3: AEE mood in third figure.

Mood

All M is P	⟶	$M\bar{P} = 0$
No M is S	⟶	$MS = 0$
Therefore, no S is P.		Therefore, $SP = 0$

After replacing the conclusion by its contradictory, we get the following antilogism:

Antilogism

All M is P	⟶	$M\bar{P} = 0$
No M is S	⟶	$MS = 0$
Therefore, some S is P.		Therefore, $SP \neq 0$.

(contradictory of E is I)

Let us examine whether the antilogism satisfy *all* the conditions or not:

(1) The antilogism satisfies the first condition. There are two equation.

$M\,\bar{P} = 0$

$M\,S = 0$

and one inequation $SP \neq 0$.

(2) But the second condition is not satisfied. The equations

$(M)\bar{P}$

$(M)S$

do not contain a common term which once occurs affirmatively and at another place occurs negatively. The common term here occurs only affirmatively at both the places (M and M). Since the second condition is not satisfied so the syllogistic mood from where the antilogism is derived, is invalid.

The student should notice that even third condition is *also not* satisfied.

Example 4:

Some politicians (P) are not married ($\bar{M}$) $\longrightarrow$ $P\,\bar{M} \neq 0$

All scientists (S) are married (M) $\longrightarrow$ $S\,\bar{M} = 0$

Therefore, some scientists are not politicians $\longrightarrow$ $S\,\bar{P} \neq 0$

After contradicting the conclusion, we get the following antilogism:

Some politicians are not married $\longrightarrow$ $P\,\bar{M} \neq 0$

All scientists are married $\longrightarrow$ $S\,\bar{M} = 0$

Therefore, all scientists are politicians $\longrightarrow$ $S\,\bar{P} = 0$

(contradictory of the conclusion O is A)

Now, let us examine whether the antilogism satisfies *all* the three conditions.

(1) The antilogism satisfies the first condition. There are two equations $S\,\bar{M} = 0$, $S\bar{P} = 0$ and one inequation $P\bar{M} \neq 0$

(2) But the second and third conditions are not satisfied. The equations

$(S)\,\bar{M} = 0$

$(S)\,\bar{P} = 0$

though have a common term but it occurs only affirmatively (S, S) at both the places. That is why the second condition is not satisfied. Notice even the third condition is also not satisfied. The inequation $P\bar{M} \neq 0$ does not possess the remaining terms of equations

$(S)\,\bar{M} = 0$

$(S)\,\bar{P} = 0$

exactly in the same form. Therefore, the syllogistic mood from where the antilogism is derived, is invalid.

Example 5 :

Some fast readers (R) are slow thinkers (T) $\longrightarrow$ $RT \neq 0$

Some children (C) are fast readers (R) $\longrightarrow$ $CR \neq 0$

Therefore, some children are slow thinkers $\longrightarrow$ $CT \neq 0$

Replacing the conclusion by its contradictory, the syllogistic mood implies the following antilogism:

Some fast readers are slow thinkers $\longrightarrow$ $RT \neq 0$

Some children are fast readers $\longrightarrow$ $CR \neq 0$

Therefore, no children are slow thinkers $\longrightarrow$ $CT = 0$

(contradictory of the conclusion I is E)

Let us see whether the antilogism satisfies all the three conditions or not

(1) The antilogism does not contain two equations. There is only one equation $CT = 0$. Since the very first condition is not satisfied, the syllogism from where the antilogism is derived, is *invalid*.

Exercise 9

Test the validity/invalidity of the following syllogistic moods by the antilogism theorem:

1.	AAA	-	3	11.	OIO	-	1
2.	EAE	-	2	12.	EAO	-	4
3.	OAO	-	3	13.	AAI	-	1
4.	AEE	-	4	14.	OAO	-	2
5.	AII	-	1	15.	EAE	-	4
6.	AOO	-	4	16.	AAE	-	1
7.	AEE	-	2	17.	AIO	-	4
8.	EIO	-	3	18.	IAO	-	3
9.	IAI	-	4	19.	OAI	-	3
10.	EAE	-	1	20.	IEO	-	4

Chapter 7 – Section D

Non-categorical Syllogism
(Compound Syllogism)

NON-CATEGORICAL syllogism is mediate reasoning having two premisses and a conclusion. But all the propositions (premisses and conclusion) are not categorical propositions in this type of reasoning. One of the propositions or all of them are compound propositions. Here are some examples of non-categorical syllogisms.

1. Either I go to his house or I will talk to him on the telephone.

   I shall not go to his house.

   Therefore, I will talk to him on the telephone.

2. If wages are high, then prices rise.

   Wages are high.

   Therefore, prices rise.

3. If Congress is united, then Congress will win the election.

   If Congress wins the election, then liberal policies will continue.

   Therefore, if Congress is united, then the liberal policies will continue.

The above syllogisms are non-categorical for they have either one or all non-categorical propositions. For the traditional logicians any reasoning having two premisses is syllogistic. But modern logicians[1] differ on this issue. For modern logicians a syllogism means only categorical syllogism. The non-categorical

1. G. Boole and those who came after him in the mid-nineteenth century.

syllogisms for them are not syllogisms at all. The examples cited above are not accepted as syllogistic reasoning by the modern logicians. In other words, whereas any reasoning having two premisses and a conclusion is called syllogistic by the traditional logicians, for the modern logicians all two premissed reasonings are not necessarily syllogistic. For the latter, syllogism means only categorical syllogism, but for the former, any two premissed reasoning is syllogistic, categorical or non-categorical. I will not enter into this controversy here. In this chapter I am dealing with different types of non-categorical syllogisms.

There are two prominent types of non-categorical syllogisms:

i. Disjunctive Syllogism

ii. Hypothetical Syllogism

Disjunctive Syllogism

Either Ram goes to library, or he will attend the seminar.
Ram does not go to library.
Therefore, he will attend the seminar.

This is an example of disjunctive syllogism. The first premiss of the syllogisms, "Either Ram goes to library, or he will attend the seminar", is a disjunctive proposition. It is a compound proposition having categorical propositions "Ram goes to library", "He will attend the seminar", which are joined by "either-or". Both of these categorical propositions are called disjuncts. The second premiss is negation of one of the disjuncts and the conclusion is remaining disjunct or disjuncts. Symbolically, the formation of disjunctive syllogism is like:

Either P or Q

not P

Therefore, Q.

Validity of disjunctive syllogism:

A disjunctive syllogism is valid when:

(i) The first premiss is a disjunctive proposition.

(ii) The second premiss is negation of one of the disjuncts of the first premiss.

(iii) The conclusion is the remaining disjunct or disjuncts.

A disjunctive syllogism is valid when it satisfies all the above conditions. The above given example of disjunctive syllogism, since satisfies all the conditions, is valid.

In symbolic form, a valid disjunctive syllogism can be one of the following types:

(i) Either p or q

not p

Therefore, q.

(ii) Either p or q

not q

Therefore, p.

An *invalid* disjunctive syllogism can be one of the following types:

(i) Either p or q

p

Therefore, not q.

Here the second premiss is not in the right form,

(ii) Either p or q

q

Therefore, not p.

Hypothetical Syllogism

In a hypothetical syllogism one or all the propositions are hypothetical. Hypothetical propositions are compound propositions combined by "if, then" relationship. For example, "If you have skill, then you will get job" is a hypothetical proposition.

Hypothetical syllogisms are of two types:

(i) Mixed hypothetical syllogism

(ii) Pure hypothetical syllogism

MIXED HYPOTHETICAL SYLLOGISM

In mixed hypothetical syllogism one of the premisses is conditional proposition having a relation of "if then" but the other premiss and the conclusion are not conditional propositions. For example,

If it is pleasant, then we shall go for picnic.

It is pleasant.

Therefore, we shall go for picnic,

is a hypothetical syllogism.

"If it is pleasant, then we shall go for picnic" is conditional or hypothetical proposition. In the above syllogism "it is pleasant" is antecedent and "we shall go for picnic" is consequent.

Since in this syllogism only one premiss is hypothetical proposition, (the other premiss and the conclusion are categorical), it is called mixed hypothetical syllogism. The second premiss in the mixed hypothetical syllogism is antecedent of the hypothetical proposition.

Validity of mixed hypothetical syllogism

A mixed hypothetical syllogism is valid when it satisfies all the following conditions:

(i) The first premisss should be hypothetical proposition.

(ii) The second premiss of the mixed hypothetical syllogism is antecedent of the first premiss.

(iii) The conclusion is consequent of the hypothetical premiss.

A valid mixed hypothetical syllogism must obey all of the above conditions. Here is an example of valid mixed hypothetical syllogism:

If I get ticket, then I will go to concert.

I get ticket.

Therefore, I will go to concert.

Symbolically, valid mixed hypothetical syllogism can be expressed as:

If p, then q

p

Therefore, q.

The rule of inference involved here is **Modus Ponens.** Ponens means to affirm; the antecedent of the hypothetical proposition, is affirmed in the second premiss; and the consequent is affirmed in the conclusion.

Valid hypothetical syllogism can also be expressed as:

If p, then q.

not q

Therefore, not p.

The rule of inference involved here is **Modus Tollens.** Tollens means to negate. The consequent is negated in the second premiss, and the antecedent is negated in the conclusion.

Invalid mixed hypothetical syllogism

Mixed hypothetical syllogism is **invalid** if it has any of the following forms:

If p, then q

not p

Therefore, not q.

or

If p, then q

q

Therefore, p.

One is not allowed to negate the antecedent in the premiss, and also one is not allowed to affirm the consequent in the premiss.

PURE HYPOTHETICAL SYLLOGISM

In a pure hypothetical syllogism, all the propositions (premisses as well as conclusion), are hypothetical propositions. For example:

If John catches the plane, then he will attend the meeting.

If he attends the meeting, then company will launch new product in the market.

Therefore, if John catches the plane, then the company will launch new product in the market.

Symbolically hypothetical syllogism can be expressed as :

If p, then q
If q, then r
Therefore, if p, then r.

Validity of pure hypothetical syllogism

A pure hypothetical syllogism is valid when:

(i) Both of the premisses have one common categorical proposition. In the above example, "he will attend the meeting", is common factor, common categorical proposition in both the premisses.

(ii) The common proposition should be antecedent in one premiss, and consequent in another premiss. The common term neutralized each other diagonally.

(iii) The conclusion should not have the common term but instead should have the antecedent of one premiss (other than common term) and consequent (other than the common term) of another premiss.

Valid hypothetical syllogisms can be expressed as:

(i) If p, then q
If q, then r
Therefore, if p, then r.

(ii) If q, then r
If p, then q
Therefore, if p, then r.

Both of the above hypothetical syllogisms are **valid.**

The following are **invalid** hypothetical syllogisms:

(i) If p, then q
If q, then r
Therefore, if r, then p.

(ii) If q, then r
If p, then q
Therefore, if r, then p.

(iii) If p, then q
If r, then q
Therefore, if p, then r.

(iv) If p, then q
If p, then r
Therefore, if q, then r.

Exercise 10

Identify the form of the following arguments and determine their validity/invalidity:

(1) If it rains in time, then there will be good harvest. It rains in time. Therefore, there will be good harvest.

(2) Either she is innocent or she is pretending. She is not innocent. Therefore, she is pretending.

(3) If the UGC stops funding the existing research projects, then there will be widespread unrest among the scholars. There is widespread unrest among the scholars. Therefore, the UGC stops funding the existing research projects.

(4) Either the coalition government wins the vote of confidence or there will be mid-term election. There will be a mid-term election. Therefore, the government will not win the vote of confidence.

(5) If the weather is pleasant, then we shall go for picnic. If we go for picnic, then we will hire two taxies. Therefore, if the weather is pleasant, then we will hire two taxies.

(6) Either Radha joins regular college or she joins the correspondence course and takes up full-time job. Radha

does not join regular college. Therefore, she joins correspondence course and takes up full-time job.

(7) Either the dictator leaves the country or he will be prosecuted. He will not be prosecuted. Therefore, the dictator leaves the country.

(8) If a person believes that everything makes for the best, then he is an optimist. He is not an optimist. Therefore, he does not believe that everything makes for the best.

(9) If Mr X gets assistance from the Bank, then he will start his business. If he starts his business, his family will prosper. Therefore, if his family prospers, then Mr X gets assistance from the Bank.

(10) If Sunny has malaria, then he needs quinine. He does not need quinine. Therefore, Sunny does not have malaria.

(11) If the patient is late for her appointment, then she has lost confidence in the doctor. If she has lost confidence in the doctor, then there is no chance of her recovery. Therefore if she is late for her appointment, then there is no chance of her recovery.

(12) If a man is happy then he is virtuous. Ram is virtuous. Therefore, he is happy.

(13) He is either intelligent or hardworking. He is intelligent. Hence, he is not hardworking.

(14) If there is a change in economic policy, then confidence in rupee will not be restored. If the confidence in rupee is not restored then imports will be restricted. Therefore, if there is change in economic policy, then imports will be restricted.

(15) If Sita goes to market, then she will buy sufficient eatables. If she buys sufficient eatables then the guests will be properly looked after. Therefore, either Sita goes to market or the guests will be properly looked after.

Chapter 8

Laws of Thought

TRADITIONALLY logic is defined as the study of laws of thought. The customary interpretation of the laws of thought is restricted to three laws, namely Law of Identity, Law of Excluded Middle and Law of Non-Contradiction.

These three laws, however, are related exclusively to propositions. Elementary logic is two-valued logic. A proposition (which is unit of reasoning) is either true or false. A proposition cannot be both true and false at the same time as well as it cannot be neither true nor false. The truth value of propositions (truth and falsehood) are **exclusive and exhaustive**. Elementary logic both traditional and modern is based on these properties of propositions. The properties of propositions are axioms. They are basis of all our valid and consistent thinking. But they by themselves are beyond proof for they are self-evident and intuitive.

The laws of thought are necessary conditions of the valid thinking. These conditions are not thoughts, they are axioms and postulates. The laws of thought are formulated variously. The most acceptable form of these laws are as follows: The Law of Identity states if a proposition is true, it is true, and if a proposition is false, it is false. If anything is A, then it is A.

P	$\supset$	P
T	T	T
F	T	F

It is a tautology. A thing implies itself. The law of identity is necessary condition of our consistent and valid thinking.

The Law of Excluded Middle states a proposition must be either true or false. It cannot be neither. If a thing is A then it is not non-A. The meaning of the law of excluded middle means middle path is not there. There is no third alternative. There are only two alternatives truth and falsehood; A and non-A. One must accept one of the alternatives and reject the other. A proposition cannot be neither true nor false.

P	V	~	P
T	T	F	T
F	T	T	F

It is a tautology. A thing must necessarily possess either of the two contradictory characteristics.

The Law of Non-Contradiction states a proposition cannot be both true and false at the same time. Out of two contradictory values true and false, a proposition can have only one value at one time. It cannot be both true and false at the same time. A thing cannot be A and non-A at the same time. If a thing is white then it cannot be non-white. If, on the other hand, a thing is non-white, then it cannot be white.

Symbolically the law of non-contradiction is stated as:

P	•	~	P
T	F	F	T
F	F	T	F

There cannot be any conjunction of two contradictory qualities.

The principle of non-contradiction (a thing cannot at the same time be and not be; a statement cannot be true and false at the same time) is the most important of the three laws. Aristotle[1] puts it at the base of all sciences. The realm of human discourse

1. Aristotle (384–322 BC) a Greek philosopher, educator, and scientist, was one of the greatest and the most influential thinkers in Western culture.

and the totality of nature all obey this pervasive law. Leibniz[2] considered the law of non-contradiction as the foundation of mathematics. This law according to Leibniz holds good for everything not only for things in this world but for all possible worlds. Thus, the law of non-contradiction is a fundamental law on which the entire logic, language and nature is based.

But the laws of thought have been criticized since the time of their formation. Some of the criticisms, however, are very trivial while some other criticisms need thorough analysis.

The standard criticism against the law of identity is that a proposition may be true and false also. For example, "It is raining" is true now but it may be false some other time. But this criticism is misleading. The proposition "It is raining" is to be seen in the matrix of space and time. If there is change in the spatio-temporal framework then the proposition "It is raining" may not be true. But as long as framework of space and time does not change the proposition "It is raining" remains true and will always remain true.

Thus the critics of the law of identity are not right. They have wrongly interpreted the proposition "It is raining". The right formulation of the proposition is "It is raining in Delhi at 2.00 p.m. on 10th July 2003" which is true and will always be true. If the reference of space and time changes then the proposition "It is raining" can be false. Each proposition has a unique reference and a fixed spatio-temporal dimension. The unique reference of space and time makes a proposition (or a fact) identical. All the propositions presuppose unconsciously this framework of space and time. In this lies the validity of the law of identity.

The principle of excluded middle is criticized on the ground that there is a third alternative for the propositions besides being true and false. A proposition may neither be true nor false; it

2. W.G. Leibniz (1646–1716) was a German philosopher, mathematician, and scholar. He and Sir Isaac Newton independently developed the theory of the differential and integral calculus.

may be a meaningless statement. For example "If Ram has killed Ravan, then 2+2=4" is neither true nor false. It is pseudo-proposition. The law of excluded middle, however, is applied only to the propositions which are meaningful. A meaningless expression is not worth considering.

Besides, it is argued there are propositions which are not fully known. It is premature to assign truth values for them. But this point is also not correct. One can wait till the unclear sentences become clear and fully understandable. One must withhold the truth values to such unclear propositions till they are fully known. Only clear and meaningful proposition is either true or false.

At the same time one must be aware of the fact that nowadays the logicians are working on the multi-valued logic which has 'n' values. It means the values of propositions can be one, two, three, four or more. But as long as we are dealing with two-valued logic, there is no third alternative. A proposition has to be either true or false.

Against the law of non-contradiction it is stated since nature changes quickly[3] (nothing is permanent) a proposition may have contradictory values. It may both be true and false. A floor is wet now but it may not be wet after some time. So a proposition has two contradictory values. But again this criticism of the law of non-contradiction is not sound. This criticism falls to ground in the same manner as the law of identity had. A proposition can have two different, rather two contradictory values in different contexts, at different times in different space. But in the same space and at the same time a proposition cannot have two contradictory values.

Besides these individual and specific criticisms against each of the laws of thought, there are general criticisms against these laws of thought as well. There are charges that these laws of

3. Heraclitus, an ancient Greek philosopher, about 500 BC says a person cannot step into the same river twice.

thought are merely trivial, or they are no more than truisms. A truism may be defined as a proposition which is (1) true, and (2) accepted by everybody on mere inspection as true. But what's the harm if these laws of thought are truism? The laws of thought are supposed to be always true and they are to be accepted by every one. Hence to charge the fundamental principles of logic with being mere truism is not to condemn them, but to admit that they are fitted to fulfil the function for which they are intended.

Along with these trivial criticisms certain genuine questions are asked regarding the validity and justification of these laws. The two common questions which are raised against these laws are as follows:

1. Are these laws exclusively for logic and language or they are applicable to things and nature also?
2. Are these laws sufficient, and complete logical principles?

The three laws of thought are applied not only to logic and language but to every realm of nature. Our entire thinking is governed by these laws and the subject matter of thinking may be any sphere of reality or nature.

Regarding the second point whether these laws are sufficient for valid thinking one can say that it is true these laws are indispensable for valid thinking, but they are not "exhaustive statement of logical principles". The principle of the syllogism, the principles of tautology, laws of inferences and laws of replacement are equally important and have "equal claim" to belong to the foundations of logic. These laws of thought are not sufficient basis from which to deduce all other logical principles. The traditional three laws of thought are indispensable but not exhaustive for valid and consistent thinking.

Chapter 9

Symbolic Logic
Its Nature and Character

ARISTOTLE, the Greek thinker, in the fourth century BC, laid the foundation of logic as a science. Since then for nearly two thousand years his work and his style of reasoning dominated the field of logic. Whatever was written by him, was considered as the Bible in logic. The medieval scholastic thinkers developed over Aristotelian logic but they adopted mainly his line of reasoning. The basic structure of logic remained more or less the same. Aristotle's contemporaries and the medieval thinkers added nothing significant to his works and they were under the impression that the last word in logic is already written by him. Some cosmetic changes were made here and there by his followers but nothing significant was added in the field of logic.

In the seventeenth century G.W. Von Leibniz, a mathematician turned philosopher, however, could see that Aristotalian and his contemporaries' style of reasoning needs modification. But he merely suggested, and never worked or showed the direction which logic possibly could take.

It is only in the nineteenth century that the thinkers started actualizing the ideas conceived by Leibniz. It is for the first time the traditional logic was seen with sharper eyes in equally sharper lights. For nearly two thousand years since its inception, logic did not progress so much as it had done in last one hundred and fifty years. This is mostly because of rapid development in mathematics and its use in logic. Since Newton's discovery of Differential Calculus, mathematics developed rapidly. Very soon, it became 'key-science'. The other branches of knowledge started using mathematical techniques in their studies. Mathematical

techniques thus started creeping into other sciences. All disciplines of knowledge started reshaping their methods and methodologies in the light of mathematical developments. Logic also joined the race.

Interestingly, all modern logicians were primarily mathematicians, and since they all were aware of mathematical techniques, it became all the more easy for them to make logic look like maths. In the beginning, it was logic which was borrowing the basic binary operations and use of variables and symbols from maths, but soon the logicians realized mathematics is actually based on logical apparatus. Logic thus became more fundamental and basic than mathematics. It started using logical properties. Modern mathematics, modern algebra, the set theory are just few examples of it. Logicians too became busy in showing logic as the foundation of mathematics. Bertrand Russell and A.N. Whitehead's *Principia Mathematica* is leading example of it.

While the traditional logicians were using non-mathematical methods to differentiate correct reasoning from incorrect ones, the modern logicians are using mathematically-oriented methods to achieve this aim. But it is not only the methods which are different in these two types of logic, modern logic is a further development of traditional logic. New avenues and fields of investigations are opened by them. The traditional logic is poorer than the modern logic in its formal character. In the classical logic the number of formula were small and so were the number of valid inference patterns. But at the same time it is not wrong to say that what was implicit in Aristotelian logic has become explicit in modem logic.

The aim of all logicians, traditional as well as modern, is to provide methods or devices to differentiate valid reasoning from invalid. Difference between classical logic and symbolic logic may be considered as a difference of degrees rather than of kinds.[1]

1. Cf. A.H. Basson and D.T. O'Connor, *Introduction to Symbolic Logic*, pp. 1-2.

For example, the difference between a big girl and a small girl is a difference of degrees and not of kind whereas difference between a girl and a boy is of kind and not of degrees. Similarly day and night differs in kind whereas the difference between morning and afternoon is of degrees. Just as a simple girl from a traditional set-up is changed in look and appearance by modern beauty tactics and she starts looking a modern girl, similarly the face of the traditional logic is lifted by modern logicians with the help of mathematical techniques. In spirit and essence it is still the same and the terminus for all logicians is to determine the validity of arguments.

Logical Form and Validity of an Argument

The rules provided by the logicians to test the validity of arguments are based primarily on the form of the argument and not on the content. Each argument is consisted of both form and matter (this was discussed in detail in the first chapter), but it is the form which is fundamental from the view point of validity. Arguments are classified according to the forms they have. Content of an argument is absolutely insignificant as far as the validity of the deductive argument is concerned.

For example any argument having either of the following forms is valid:

(i)	All M is P	(ii)	If p then q
	All S is M		p
	Therefore, all S is P		Therefore, q

Whatever may be the value of S, P, M or p and q, argument bearing either of the above forms is always valid. On the other hand, the argument having any of the following forms is always invalid:

(i)	All P is M	(ii)	If p then q
	All S is M		q
	Therefore, all S is P		Therefore, p

Whatever may be the value of S, P, M or p, q argument having any of the above form is always invalid.

The symbols like S, P, M or p, q are variables. They are "dummies" in place of which any desired statements may be imagined.[2] They keep changing their values from argument to argument. By giving different values to S, P, M or p, q innumerable arguments can be constructed having similar form though different content. Rules and formulae to decide the validity of arguments are made according to the form they have, and not according to their content.

For this reason, all the logicians specially deductive logicians, both ancient as well as modern, are interested in the form of an argument. The entire show in the deductive logic is of forms, and because of this deductive logic like mathematics is called formal science.

However logic, "seems to differ from mathematics in that in logic we talk about statements and their interrelationships, notably implication, whereas in mathematics we talk about abstract non-linguistic things: numbers, functions, and the like".[3] Yet logic sometimes makes use of the mathematical jargons like law of association, law of commutation, law of distribution, De Morgan's laws, etc. about which you will study later on.

Advantages of Using Symbols

In sharing the same platform with mathematics, modern logicians made extensive use of symbols. The classical logic used symbols like S, P, M which denote minor term, major term and middle term in a categorical syllogism, respectively. Also A, E, I, O symbols were used to denote various categorical propositions. The modern logicians extended the use of symbols in logic. The phrase 'symbolic logic' itself suggests their predomination in logic.

2. W.V. Quine, op.cit., p. 29.
3. Ibid., p. 5.

Two types of symbols are used — variable and constant. A variable symbol keeps on changing its value from argument to argument. These symbols are "dummies". They do not have fixed values. For example, P in one argument may stand for "You will pass", in another argument P may represent. "It is very pleasant today" and yet in third argument it may be assumed for "We will go for picnic". Here symbol P is variable. Many types of variables are used in modern logic such as propositional variables, predicate variables, class variables, free and bound variables and many more of its type. In the traditional logic only three variable symbols were used, that is, S, P and M for the various terms of a categorical syllogism. This suggests that the use of variable was known to Aristotle though it was restricted only to a few symbols.

However, use of symbols for constants was done for the first time by the modern logicians. Constant symbols do not change their value throughout the domain of logic. Some important constant symbols are as follows:

- $\sim$ for negation
- v for the relation of "either or"
- $\bullet$ for the relation of 'and'
- $\supset$ for "if then" relationship

A question may arise why the modern logicians make extensive use of symbols? What are the advantages of using symbols in logic? Symbols are of big use to a logician at least in two ways. Firstly, the logical form of an argument becomes explicit by using symbols. The complexity of language may obscure the structure of an argument, and this can conceal its logical form. Not only in logic, language even otherwise is considered a villain in philosophy. The use of language in logic can even be more nasty as slightest variation in the interpretation of terms or propositions can change the form of reasoning. Replacing language by symbols the logical form of an argument becomes explicit. Once right logical form of an argument is captured, its validity can be decided quickly and accurately.

Secondly, the use of symbols in logic is seen as an economical device. The long and big arguments become small and handful expressions after symbolization. They then can be operated quickly and easily. The chances of committing errors in deciding their validity are thus reduced.

Not only in logic but even otherwise there is an increase in use of symbols and code language nowadays. All sciences, social as well as natural, make use of more and more symbols for being precise and accurate.

Inference and Implication

The chief characteristic of deductive logic is that it is impossible to have all true premisses and a false conclusion. In other words, in a valid deductive argument if all the premisses are true, then the conclusion must necessarily be true. However, this relationship between premisses and conclusion in logic is expressed in two ways, inference and implication. Though sometimes the difference between them is very thin, nonetheless, the difference is there.

The inference is made *by person,* but it is *propositions* which imply. In inference, the agent, the man believes in the truth value of premisses, and because of this he believes in the truth value of the conclusion. For example, when I say:

All men are mortal.
Socrates is a man.
___
Therefore, Socrates is mortal.

I assert the premisses and thus I assert the conclusion. In order an inference is logically sound or logically justifiable, it must satisfy two conditions:

(i) the inferer believes in the truth value (truth and falsity) of the premisses.

(ii) the conclusion follows from the premisses in accordance with the logical rules. In other words, the premisses and the conclusion are logically related to each other.

But in order an implication is logically valid, it must satisfy just one condition, that is, the relation between antecedent (premisses) and consequent (conclusion) should be logically acceptable. The second condition of true inference is the only condition which applies to valid implication.

An implication is totally formal. In implication, the logician may or may not believe in the truth value of the antecedent (premisses) and consequent (conclusion). The logician may also not believe that the meaning of antecedent and consequent are related to each other. For example, "If Ram has killed Ravan, then 2+2 = 4" is logically true implication in spite of the fact that the meaning of the premiss (antecedent) is not related to the conclusion (consequent) at all. Whereas the former is mythical proposition, the latter is a mathematical one. Thus is order that an implication is true or logically acceptable, it must satisfy only the logical rules, i.e. if one proposition is true, each proposition "implied by it must also to be held as true". The logicians' belief or disbelief in the antecedent plays no role in evaluating an implicative relation.

The formats of inference and implication are also different. For example:

All men are mortal.
Socrates is a man.
___
Therefore, Socrates is mortal.

is an inference. The same content can be expressed in implication as follows:

If all men are mortal and Socrates is a man, then Socrates is mortal.

An inference is expressed in the form like: Such and such premisses *therefore* the conclusion. Implication is expressed as: if such and such propositions *then* this proposition.

An inference can be expressed in implication and an implication can be expressed in inference after changing their

formats. **Whereas in the traditional logic there is extensive use of inferences, the modern symbolic logic makes use of implication only. This is a big difference.** This highlights that the traditional logic is not totally formal whereas the modern logic is completely formal in character.

Chapter 10

Symbolization

THERE are twofold tasks in the elementary symbolic logic for the beginners. They are:

(i) Symbolization of propositions and arguments stated in ordinary language.

(ii) Testing the validity of various types of arguments, evaluating, whether statement expressions are tautologous or not, and also construction of demonstration showing whether a statement implies or is equivalent to another statement.

This chapter is concerned with the first of these tasks, that is, with symbolization. For a simple proposition which is a simple sentence also, one letter from alphabet is assigned. For example:

Ram is graduate, is symbolized as G
Sita is good student, is symbolized as S
New Delhi is capital of India, is symbolized as C

Rita is intelligent and she is good student, is symbolized as: I and S.

If it rains, then we shall go for picnic, is symbolized as: If R, then G.

Either I will go to library or attend the seminar is symbolized as: Either G or A.

You will get job if and only if you qualify this test, is symbolized as: G if and only if Q.

Negative proposition: Propositions like "Ram is not a good student", "It is not pleasant today", "We shall not attend the seminar", etc. are negative propositions. They are symbolized as follows:

(i) Ram is not a good student

~ S

S - Ram is a good student

~ - not

(ii) It is not pleasant today

~ P

P - It is pleasant today

~ - not

(iii) We shall not attend the seminar

~ A

A - We shall attend the seminar

~ - not

"~" is sign for "not" and is read as tilde or curl.

There are other words and phrases besides "not" for which "~" sign is substituted, such as "it is not the case", "it is false", "it is not true", "it is untrue".

It is not the case that the indiscipline is tolerated = ~ T
It is false Congress has won the assembly election = ~ W
It is not true that an honest man is a man of millions = ~ M
It is untrue that Indians are indisciplined citizens = ~ I

However, symbolization of propositions like "It is true Indians are religious minded people" is P.

Symbolization of Compound Propositions

Simple propositions are combined by various relations to form compound propositions.

Compound propositions are of various types: conjunctive, disjunctive, conditional (implicative) and bi-conditional (equivalence).

1. CONJUNCTIVE

In conjunctive proposition, two simple propositions are joined by "and". For instance, Ram is intelligent and he is a hardworking student. This is symbolized as follows:

I• H

I Ram is intelligent

H He is hardworking student

"•" is a constant symbol used for "and" and is read as "dot".

There are many other words besides "and" for which "•" sign is used. These words are: but, yet, both, although, however, moreover, neither nor, as well as while, etc. For example:

He is poor but he is honest

P•H

It is hot yet tolerable

H•T

He is intelligent although not very careful

I•~C

Both Mohan and Rohit are students of this college

M•R

Not both Sita and Radha attended the convocation

~ (S•R)

But *both not* is symbolized differently.

A and B both were not selected for the job

~A•~B

Conjunctive sign "•" is used for "neither nor" relationship also. "Neither Radha nor Mohan are graduates" is symbolized as ~ R•~ M. This means Radha is not graduate and Mohan is also not graduate.

2. DISJUNCTIVE

Two simple propositions when combined by "either or" is called disjunctive proposition. "Either I will telephone him or I will write to him", is symbolized as:

T v W

T - I will telephone him

W - I will write to him

"v' - "either or'

"v" sign shows the relationship of "either or". It is read as "wedge". "Either it rains or we shall not go for picnic", is symbolized as:

R v ~ G

R - It rains

~G - We shall not go for picnic

v - sign for "either or" and is constant.

Either Indians are coward or they are god fearing.

C v G

C - Indians are coward

G - God fearing

v - "either or".

Either A is not honest or B is not telling truth.

~H v ~T

If two propositions are joined by "unless" then also disjunctive sign can be used. For example:

I do not go to market unless it is very necessary, is symbolized as:

~G v N

-G - I don't go to market

N - It is very necessary,

v - disjunctive sign

Take another example: Unless you work hard you will not pass, is symbolized as:

~P v H

H - You work hard

~P - You will not pass

v - disjunctive sign

"Neither nor" relationship to join two simple propositions can also be symbolized by using "v" sign. For example:

Neither soldiers nor officers are demoralized by this incident.

~(S v O)

S - Soldiers are demoralized

O - Officers are demoralized

Earlier "neither nor" relation was symbolized by using "•" sign. The above proposition can also be symbolized in terms of "•" like:

~S•~O

Thus,

~(S v O) ≡ (~S• ~O)*

"Neither Ram nor Mohan was elected in the assembly election" can be symbolized in two ways:

(i) ~R• ~M

and

(ii) ~(R v M)

3. IMPLICATION

Simple propositions when combined by "if, then" relation is called hypothetical, conditional or implicative proposition. When two propositions are joined into an implicative proposition then the first is said to imply the second, or second is implied by the first.

A conditional or implicative proposition is symbolized by "⊃" and is called "horse shoe" for it looks like shoe of a horse.

If it rains, then we shall go for picnic.

R⊃G

R - It rains

G - We shall go for picnic

⊃ - sign of "If then"

If it does not rain, then we shall not go for picnic, is symbolized as:

~R ⊃ ~G

In the "p ⊃ q" statement form p is "antecedent" and q is "consequent". While symbolizing an implicative proposition, it is

* More such bi-conditionals are discussed in the next chapter.

important to identify the correct antecedent and correct consequent. For instance in the proposition.

"You will get job only if you qualify the test", is symbolized as:

$G \supset Q$

You will get job	-	G	-	antecedent
You qualify the test	-	Q	-	consequent

You will get job if you qualify the test, is symbolized as:

$Q \supset G$

You qualify the test	-	Q	-	antecedent
You will get job	-	G	-	consequent

All the following statement forms are symbolized as $p \supset q$:

p only if q
q if p
q provided that p
q on condition that p
q in case p
p hence q
p implies q
Since p, then q
"q is a necessary condition for p"
"p is a sufficient condition for q"
"only if q, p".

4. EQUIVALENT OR BICONDITIONAL PROPOSITION

Bi-conditional compound proposition is obtained by combining two simple propositions with equivalent relation. For example "You will catch plane if and only if you reach in time" is equivalent or bi-conditional proposition, and is symbolized as:

$C \equiv R$

C	-	You will catch plane
R	-	You will reach in time
$\equiv$	-	symbol used for "if and only if" relation

You will get job if and only if you qualify the test.

$G \equiv Q$

In English language "if and only if" phrase stands for equivalent relation between propositions, and is symbolized as "=".

Look at the following compound propositions having similar but logically different forms:

(i) Mr. X will catch bus if he reaches in time
$R \supset C$

(ii) Mr. X will catch bus only if he reaches in time
$C \supset R$

(iii) Mr. X will catch bus if and only if he reaches in time

$C \equiv R$

R	-	Mr. X reaches in time
C	-	Mr. X catches the bus

One has to be very careful in symbolizing the propositions. For a slight variation in the symbolization results in wrong evaluation of the truth value of statements and arguments.

Now let us see how **arguments are symbolized.** Consider the following arguments.

Example 1 : He is either insecure or he is guilty. If he is guilty, he must tell everything to police. He will not tell everything to police. Therefore, he is insecure.

The following symbols are substituted:

He is insecure	-	I
He is guilty	-	G
He must tell everything to police	-	T
1st premiss	-	$I \vee G$
2nd premiss	-	$G \supset T$
3rd premiss	-	$\sim T$
Conclusion	-	I

In a argument one premiss means one complete sentence. Full stop marks the completion of a sentence. The sentence with any of the words, "therefore", "thus", "hence", "so" is conclusion.

Example 2 : If I watch late night TV then I will not study for the test. If I do not study for the test, then tomorrow either I will miss my class or I will have to take an off. I will not miss my class. Therefore, I will study for test.

Symbolization:

I watch late night TV	-	W
I will not study for test	-	$\sim S$
I will miss my class	-	M
I will have to take an off	-	T
1st premises	-	$W \supset \sim S$
2nd premiss	-	$\sim S \supset (M \vee T)$
3rd premiss	-	$\sim M$
Conclusion	-	S

Example 3 : A car gives good service if and only if it is properly maintained. If this car is properly maintained, then it consumes *less* petrol. This car consumes *more* petrol. Therefore, this car is not properly maintained.

Symbolization:

Car gives good service	-	G
Car is properly maintained	-	M
Car consumes less petrol	-	C
1st premiss	-	$G \equiv M$
2nd premiss	-	$M \supset C$
3rd premiss	-	$\sim C$
Conclusion	-	$\sim M$

One important point which the student should notice is that there is difference between

(i) *argument* and *argument form*

(ii) *statement* and *statement form*

The above given examples are of arguments. They are concrete, individual arguments. The capital letters of the alphabets are used to symbolize them. The argument form, however, is an abstract arrangement of premisses and conclusion in which "dummies" are used for premisses and conclusion. Countless arguments can be obtained from the same argument form. For instance.

$p \supset q$
$q \supset r$
Therefore, $p \supset r$

is an argument form and p, q and r are propositional variables and are no more than "dummies". The argument forms are symbolized by using small letters of the alphabets whereas arguments are symbolized by using capital letters.

Similarly the statement form is an abstract arrangement of the prepositional variables. A statement form is symbolized by using small letters of the alphabets. For instance $(p \supset q) \vee p$ is statement form and p, q are prepositional variables whereas, "He is poor yet he is honest" is statement symbolized as $(P \bullet H)$. A statement form can be replaced by statement, and also argument form by argument. In doing so the prepositional variable p, q, r are replaced by prepositional symbols P, Q, R, etc.

Exercise 11

(A) Symbolize the following statement forms by using suitable abbreviations:

1. a if b.
2. a only if b
3. a if and only if b
4. Either a or not b.
5. Both a and b
6. Either not a or b
7. a if and only if not b

8. If a, then b and c
9. If a then b, and c
10. If either a or b, then c
11. If a, then either a or b
12. a, if b and c
13. If not a, then b and c
14. If both a and b, then both c and d
15. a implies b, and c implies d.
16. a provided b.
17. not p unless q
18. unless p, not q
19. It is not the case that not a and not b
20. It is not the case that a, and not b
21. It is not the case that if a, then b
22. It is not the case that if a then b
23. It is not the case that either a or b
24. It is not the case that if a, then b and c
25. It is not the case that if a then b, and c
26. It is not the case that both a and not b
27. It is not the case that either not a or b
28. Either both a and b or neither a nor b
29. Either not both a and b or both not a and b
30. p implies q if and only if q implies p.

(B) Symbolize the following statements:

1. She attended to her duties and earned promotion.
2. They saw the uselessness of violence yet did not change their policies.
3. I sit on the chair but my cat sits on the floor.
4. If he runs at top speed, then he will get out of breath.

5. Either he leaves the country or he is prosecuted.
6. Mr. X will be prosecuted unless he co-operates with the investigating team.
7. Two triangles are formed if a square is divided diagonally.
8. He was annoyed, still he kept quiet.
9. He failed though he tried.
11. Be neither a barrower nor a lender.
12. I cannot drive a car, if it is dark.
13. A child is suspended from the school if his behaviour is indecent.
14. He is both fool and coward.
15. Unless we run, we shall miss the train.
16. He escaped several times but was finally caught.
17. Either the principal did not notice the change or else he approves of it.
18. We will go if it does not rain.
19. A wins only if B does not contest.
20. He is neither hardworking nor honest.
21. It is not the case that he is hardworking and he is honest.
22. Unless it rains in time, harvest will not be good.
23. It is not the case that if any metal is heated, it will not expand.
24. It is false that the guard raised his gun yet could not scare away the thieves.
25. He is a good student therefore he is loved by all.
26. It is not true that he reads *Patriot* but not *Reader's Digest*.
27. It is not true that either James did not win or that John did not win.
28. It is not true that neither James won nor John won.

29. A will dance only if both B and C sing.
30. A will dance if B sings and C also sings.

(C) Symbolize the following statements:

1. S will go to college if and only if K also goes to college.
2. Either she is too good and innocent, or she is pretending to be so.
3. If A and B both do not win then not both C and D win.
4. K will go to market and B will also go to market, if S goes to school.
5. K and B go to market only if S goes to school.
6. It is not true that she goes to market but not buy anything.
7. If mathematics is difficult, then you will pass only if you work hard.
8. Rohit will get job only if Mohan does not, but it is not the case that both Rohit and Mohan get job.
9. Either A and B both go to library or neither will.
10. If the wind blows fast, then tents will be uprooted and soldiers will have bad time.
11. If the boats are made of bark, then they are very light and can easily be carried on the shoulders.
12. Either make haste and get ready soon or you will be late for the meeting.
13. If I do not go to Bombay, then I will meet my dentist and fix an appointment for the operation.
14. If the police arrives in time and starts firing, then the crowd will thin out.
15. If hydrogen and oxygen are mixed in the required proportion, water is produced.
16. Siddharth is a big boy and strong also, still he was not selected for the marathon race in the school.

17. If rain falls steadily for several days and the river overflowed its banks, then the terrified villagers would abandon their homes and would fly to the higher grounds.
18. Either it is false that monsoon failed and tanks become empty, or grains could not be sown and famine is feared.
19. If an author is genius, then he suffers the penalty of genius, and if he has talent, then various cares and worries make his life extremely miserable.
20. The Suez Canal can neither to be obstructed nor closed, if treaty is signed among the principal nations of Europe.
21. Either French and Italian languages are different forms of Latin or it is false that they have different origin.
22. If I stay home and watch the late show, then either I do not do my homework or go for a long walk.
23. Allen will get job only if Brown does not, but it is not the case both Allen and Brown get job.
24. If the weather is pleasant and there is a holiday, then either we go for picnic or it is not the case that we go to market.
25. If Sita goes to the party if and only if Radha goes to the party, then either both will go to the party or neither will.

(D) Symbolize the following arguments:

1. If John is healthy, he is happy. He is happy. Therefore, John is healthy.
2. If a metal is gold, then it is yellow. This metal is gold. Therefore, it is yellow.
3. This fellow is either a fool or a fraud. He is not a fool. Therefore, he is a fraud.
4. Either a disease is hereditary, or it is contagious. Tuberculosis is contagious. Therefore, tuberculosis is not hereditary.

5. The universe is either contingent or self-explanatory. But the universe is not self-explanatory. Therefore, the universe is contingent.
6. If A goes to the market, then B will do baby sitting. If B does baby sitting, then C goes for a movie. Therefore, if A goes to market, then C goes for a movie.
7. Unless a person believes that everything makes for the best, he is not an optimist. A person either believes that everything makes for the best, or he does not believe in that. Hence, a person is either optimist or he is not.
8. If a child is interested in studies, then he will pay attention and will enjoy studies. If a child does not pay attention to studies, then he will not learn. The purpose of teaching is defeated if the child does not learn. So if the child does not enjoy studies then the purpose of teaching is defeated.
9. If the police obeys the laws, then they will put an end to smuggling. But if they keep their job, then they will not put an end to smuggling. Thus, they either do not obey the laws or do not keep their job.
10. It is false that species of animals are fixed in number and that new species are not formed. But we know that they are not fixed. It implies that new species are formed.
11. If the doctor knew that the speed limit is forty kilometre per hour, then he would not have been driving at sixty. The doctor did not know the speed limit is forty kilometre per hour. He was driving at sixty kilometre per hour and was caught by the traffic police. Hence either the doctor did not know the speed limit is forty kilometre per hour, or he was caught by the traffic police.
12. If life exists on Mercury, then its temperature should neither go above boiling point nor should go below freezing point. Mercury is very lose to the sun. Its temperature (on the day light side) goes above boiling point and its temperature (on its opposite night side) goes below freezing point. Hence life cannot exist on Mercury.

13. If men were totally unjust, then they differ in their opinions and they could not act together in harmony to carry out a plan. Robbers act together in harmony to carry out a plan. Therefore, robbers are not totally unjust.
14. A student is caught copying if and only if the invigilators are alert. If the invigilators are alert and government makes strict anticopying laws, then the malaise from the education sector can be rooted out. But neither the invigilators are alert nor the strict anticopying laws are there. Hence, the malaise from the education sector cannot be rooted out.
15. If it is false that X's passing the examination implies that he is lucky, then, if X gets good marks, then the examiner was impartial. Therefore, if X passed the examination and he gets good marks, then, if the examiner was impartial, X was not lucky.

Chapter 11

Truth Function

We have seen in the previous chapter symbols like "•", "⊃", "v", "≡" are very important. They connect simple propositions to form compound propositions. They are thus called logical connectives. Compound propositions using these logical connectives are truth functionally compound. Truth functional logic is called "logic of statement connections". Conjunction, disjunction, implication, bi-conditional, etc. are logical connectives for they connect two simple propositions.

In the elementary symbolic logic seven types of truth functions are recognized. They are:

1. Negative
2. Conjunctive
3. Disjunctive
4. Alternative
5. Implicative or conditional
6. Equivalent or bi-conditional
7. Stroke.

Before each one of them is discussed in detail, let us first understand the meaning of function and truth function. The word "function" is a familiar notion in mathematics. When the value of an expression depends on the value of its constituents, then the expression is said to be function of its constituents. For example in the expression:

$y = 4x + 8$

the value of y depends on the value of x. The moment x is given a value, the value of y becomes known. If x is 3 then:

$y = 4x + 8$
$y = 12 + 8$
$y = 20$

If the value of x changes then value of y also changes subsequently. In that case y is called function of x.

In the expression

$z = 3x + 2y + 5$

z is function of x and y. The value of variable z depends on the values of two variables x and y.

In logic the word "function" is used exactly in the same sense as in mathematics. But instead of using word "function", the logicians preferred "truth function". There is a reason for this. The logicians are interested in the truth value of expressions and validity of arguments. The logicians are looking for truth conditions of statements and statement forms.

A proposition is called truth functionally compound when its value depends on the truth value of its components and connectives. For example, the value of the expression "p•q" depends on the value of "p", "q" variables. Thus, "p•q" is truth function of "p", "q". The truth value of the proposition "Ram is honest and Ram is hardworking" depends on the truth value of "Ram is honest" and "Ram is hardworking".

Let us examine the seven types of truth functions one by one in detail.

1. Negative Function

The negation of a proposition is another proposition whose value depends on the original proposition. The negation of the proposition "Cats are mammals" is "Cats are not mammals". If the former proposition is true, the latter is false. In other words, if p is true, then not p is false; if p is false, then not p is true. To deny a proposition is to affirm another proposition known as the negation N or contradictory of the first.

p	~p
T	F
F	T

Since the value of ~P depends on the value of p variable, ~p is truth function of p. Double negating a proposition means to affirm a proposition. ~ ~p = p.

Negative function is very important for the interdefinability of truth functions (logical constants). Later in the chapter this is discussed in detail.

2. Conjunctive Function

Two simple propositions when combined by "and" makes a compound proposition called conjunctive proposition. For example, "It is very hot and it is very humid" is a compound proposition having a conjunctive sign.

Symbolically a conjunctive proposition is written like p•q. The truth value of a conjunctive proposition depends on its constituents. The truth value of the proposition "p•q" depends on the value of "p" "q" and "•". Since the value of "p•q" depends on the variable "p", "q" and the characteristics of "•", it is called conjunctive function.

The constituents of conjunctive proposition p, q are called conjuncts. A conjunctive proposition is true when all its conjuncts are true. A conjunctive proposition is false if one of the conjuncts is false.

p	q	p•q
T	T	T
T	F	F
F	T	F
F	F	F

The table shows conjunctive function "p•q" is true when both of its conjuncts p, q are true. If either of conjuncts p, q is false, then conjunction becomes false.

Let us see some solved examples here:

1. New Delhi is capital of India and Dr. Rajendra Prasad was the first President of free India.

   T and T
   T•T
   T

2. Cats are mammals and fish are also mammals.

   T and F
   T•F
   F

3. Mrs. Gandhi was 5th President of India and India is the biggest democratic country in the world.

   F and T
   F•T
   F

4. Children are voters in India and India is in Australian Continent

   F and F
   F•F
   F

Sometimes values of the variables are given, and on that basis the value of the expression can be worked out. For example, if A, B are given as false, then the following expression is true.

~ (A•B)
= ~ (F•F)
= ~ F
= T

Conjunctive function has other important characteristics also which can be explained by using "mathematical jargons". Conjunctive function is *commutative, associative* as well as *idempotent.*

$p \bullet q \equiv q \bullet p$	Commutative property
$p \bullet (q \bullet r) \equiv (p \bullet q) \bullet r$	Associative property
$p \bullet p \equiv p$	Idempotent property

In certain cases, the value of one conjuct or more conjuncts are unknown. As for instance if p is unknown then the values of the following expressions are as such:

(i) $p \bullet T = p$

(ii) $p \bullet F = F$

(iii) $p \bullet \sim p = F$

Here the value of p variable is not known yet in some cases one can find the truth value of conjunctive compound propositions. The truth value of the expression (i) p•T depends on p. If p is true then p•T is true; similarly if p is false, p•T is false. That is why p•T is equivalent to P.

The truth value of the expression (ii) p•F is false. For a conjunctive function is true when all conjuncts are true. Here one of the conjuncts is false, hence p•F is false.

Similarly, the truth value of the expression (iii) p•~p is false because whatever may be the value of p, ~p is the contradictory of it. Hence one of the conjuncts (either p or ~p) will be false. Thus the expression p•~p is false.

3. Disjunctive Function

Two simple propositions when combined by "either or" makes disjunctive proposition. "Either Sohan is ill or he is involved in some important matter" is disjunctive proposition symbolized as I v M. Both "I" and "M" are disjuncts.

The truth value of a disjunctive proposition "p v q" depends on the values of its disjuncts p, q and the characteristic of "v". Thus, the proposition "p v q" is truth function of "p", "q".

A disjunctive proposition is true when at least one of the disjuncts is true. When all the disjuncts are false, then the disjunctive proposition is false.

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

Some solved expressions are given below. If p is true and q is false, then the value of the following expressions are worked out as follows:

1. $\sim (p \vee q)$
$= \sim (T \vee F)$
$= \sim T$
$= F$

2. $(p \vee q) \vee (p \bullet q)$
$= (T \vee F) \vee (T \bullet F)$
$= T \vee F$
$= T$

3. $\sim (p \bullet q) \vee (p \vee q)$
$= \sim (T \bullet F) \vee (T \vee F)$
$= \sim (F) \vee T$
$= T \vee T$
$= T$

4. $\sim (p \vee \sim q) \vee (q \bullet \sim q)$
$= \sim (T \vee \sim F) \vee (F \bullet \sim F)$
$= \sim (T \vee T) \vee (F \bullet T)$
$= \sim T \vee F$
$= F \vee F$
$= F$

5. $\sim (p \bullet p) \bullet \sim (q \vee p)$
$= \sim (T \bullet T) \bullet \sim (F \vee T)$
$= \sim (T) \bullet \sim (T)$
$= F \bullet F$
$= F$

Like conjunctive function disjunctive function also has certain mathematical properties. Disjunctive function has *commutative, associative* as well as *idempotent* properties.

$p \vee q \equiv q \vee p$	Commutative property
$p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative property
$p \vee p \equiv p$	Idempotent property

Besides, the value of disjunctive propositions in certain cases can be known even if all the disjuncts are not known. For instance if p is unknown the value of following statement forms are as follows:

(i) $p \vee T = T$

(ii) $p \vee F = p$

(iii) $p \vee \sim p = T$

The value of p variable is not known yet in certain cases the truth value of disjunctive compound propositions can be determined. The truth value of (i) expression p v T is true for a disjunction is true even if one of the disjuncts is true. The truth value of (ii) expression p v F is p. Whatever will be the value of p, p v F expresion will have that value. If p is true, pvF is true and, on the other hand, if p is false pvF is false. The truth value of (iii) expression pv~p is true, for whatever may be the value of p, one of the disjuncts in pv~p will be true. Thus pv~p in any case will be true only.

Both conjunctive and disjunctive functions resemble certain mathematical binary operations like addition and multiplication (but not substraction and division) in the sense that they have associative and commutative properties.

However, the relationship of "either or" is not a simple one. In logic this relationship is interpreted in two different ways: one is disjunction and the other is alternation. The following examples may help to see the difference:

A mother asked her daughter: choose one of the dresses which I selected for you. Either take dress A or B. The mother obviously intends the child should pick up one and reject another. But she is not surprised when the daughter says I will take both. The example shows one can accept both the options A and B. This is disjunctive sense of "either or".

Take yet another set of example: suppose there is a guest and I ask him what will you take, tea or coffee? I am certainly surprised if the guest says both. Here "either or" is used in the alternative sense. The guest is given option to choose one out of two alternatives. If a professor asks a student what do you want to do? You want to study or you want a job. The professor is quite comfortable when the student says I want to do both. This is the use of disjunction. Alternative means not both. It means p or q but not both.

There are two alternatives. One is to be accepted and the other is to be rejected. One is not allowed to accept both, and one is not allowed to reject both either. This interpretation of "either or" is called alternative, and is symbolized as $p \wedge q$.

4. Alternative Function

In the alternative function, if one of the alternates is true and the other is false, only then it (alternative function) is true.

p	q	$p \wedge q$
T	T	F
T	F	T
F	T	T
F	F	F

Let us compare alternative function with disjunctive function.

<u>*Disjunctive function*</u>
(inclusive sense of "or")

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

<u>*Alternative function*</u>
(exclusive sense of "or ")

p	q	P^q
T	T	F
T	F	T
F	T	T
F	F	F

The difference between disjunctive and alternative interpretations of "either or" lies in the first row. Whereas the disjunctive function allows all the disjuncts to be true, the alternative function does not allow all the alternates to be true. Disjunction is weaker and inclusive sense of "either or" whereas alternation is stronger, strict and exclusive sense of "either or".

However, while symbolizing the arguments in the elementry symbolic logic, only disjunctive function is used for "either or" relationship although in our daily life, we use "either or" in both the senses.

5. Implicative Function

In an implicative function two simple propositions are combined by "if then" relationship. For example, "If it is pleasant then we shall go for picnic" is conditional proposition. It is symbolized as P ⊃ G. Here P is antecedent and G is consequent. The truth value of "p ⊃ q" depends on the truth value of antecedent "p" and consequent "q" and "⊃". Thus "p ⊃ q" is truth function of "p" and "q".

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

An implicative function is false when antecedent is true and consequent is false. Incidentally this is the chief characteristic of deductive logic also. If all the premisses are true and conclusion is false, then a deductive argument is invalid. A true proposition can imply only a true proposition.

Let us see some solved examples:

If p is true and q is false, the values of the following expressions are worked as follows:

1. (p ⊃ q) v p
   = (T ⊃ F) v T
   = F v T
   = T
2. q ⊃ (p•~q)
   = F ⊃ (T•~F)
   = F ⊃ (T•T)
   = F⊃T
   = T
3. ~ (p•q) ⊃ ~q
   = ~ (T•F) ⊃ ~F
   = ~F ⊃T
   = T ⊃ T
   = T

"If then" relation like "either or" is not a simple one. It has been interpreted differently by logicians and common men. A common man relates two propositions by "if then" only if the meanings of the propositions are also related. For example, the proposition, "If Ram has killed Ravan, then 2+2=4" is neither true nor false expression for a common man. It is rather a meaningless, a pseudo expression. But to a logician the above implicative expression is true. For a common man the example given above is pseudo expression because the meanings of the propositions are not related. Whereas one proposition is mythical, the other is arithmetical. But it is not only the meanings of the propositions which are same but consequent also seems to follow

from the antecedent in the ordinary understanding of "if then" relation. In the expression, "If it rains, then we shall go for picnic", "our going" depends and follows from the proposition "it rains".

Thus, for a common man two simple propositions are related by conditional relation only when their meanings are related, and also when the consequent follows from the antecedent. But for a logician any two simple propositions can be combined by "if then" whether their meanings are related or not, provided antecedent should not be true if consequent is false. A logician is interested only in the logical properties of an implicative relation, and not in the meaning of its constituents.

Two different standpoints (ordinary and logical) were probably the main stumbling stone in the formation of the truth value of a implicative function. The common use of "if then" relation provides no clue in formulating the truth value of implication. Its truth value was thus formulated indirectly by taking help of other two functions, negation and disjunction. For instance, "If it rains, then we shall go for picnic" can be rewritten as "Either it does not rain or we shall go for picnic" without distorting the meaning of the original proposition.

$p \supset q$ has the same truth value as $\sim p \vee q$

p	~ p	q	≡ pvq	≡ p ⊃ q
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T

The truth value of implication is thus determined indirectly. Still the problem is not over for the implication because there appears to be twofold paradox due to its truth value.

PARADOX OF MATERIAL IMPLICATION

If we look at the truth value of implicative function there seems to be twofold paradox.

The truth value of implication is as follows:

p	⊃	q
T	T	T
T	F	F
F	T	T
F	T	F

The first row states that a true proposition implies a true proposition.

The third row states that a false proposition can imply a true proposition.

Two together state: A proposition true or false can imply a true proposition. In other words: any proposition, true or false, can imply a true proposition. This is the result of the logical properties of an implication. But to a common man this result (that any proposition, true or false can imply a true proposition) is unacceptable. A true proposition implying true proposition is accepted; a false proposition implying false proposition is also accepted. But how can any proposition (true or false) imply any true proposition? This is paradoxical situation.

Now let us see how the second paradox occurred.

The third row states that a false proposition can imply true proposition.

The fourth row states that a false proposition can imply false proposition.

Together they state: Any proposition true or false can be implied by a false proposition. This is the logical outcome but again to a common man this is unacceptable. A false proposition implied by false proposition is accepted; but how can any proposition true or false be implied by any false proposition?

There is reason behind these paradoxes. The ordinary use of "if then" did not help the logicians to construct the truth values of implicative function, though in all other relations like

conjunction, disjunction, negation, alternation and also equivalence (about which you will study later on) the ordinary language helped in framing their truth values. The truth value of implication was established indirectly, and since it does not fit with ordinary use of "if then", there is clash and hence paradox. The twofold paradox of *material implication* can be resolved or dissolved if the logical standpoint is kept separate from the ordinary standpoint. If we allow them to mix, the paradox is imminent. However, if they are kept separate, no paradox is there. In order to make things clearer and better the logical stand point of "if then" is called *"material implication"*.

6. Equivalent Function

Two propositions or expressions are equivalent if they have same truth value. If both the propositions are true then they are equivalent and similarly if both the propositions are false then also they are equivalent. However, if one proposition is true and the other is false, then they are not equivalent. The truth value of equivalent proposition can be framed with the help of ordinary understanding of the word equivalent.

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Since the truth value of "$p \equiv q$", depends on the truth values of "p", "q" the expression "$p \equiv q$" is truth function of its constituents "p", "q".

Let us see some solved examples. If p is true and q is false, then values of the following expressions are worked as follows:

1. $p \supset (q \equiv p)$
$= T \supset (F \equiv T)$
$= T \supset F$
$= F$

2. $\sim p \equiv (p \supset p)$
$= \sim T \equiv (T \supset T)$
$= F \equiv T$
$= F$

3. $(p \supset q) \equiv (\sim p \vee q)$
$= (T \supset F) \equiv (\sim T \vee F)$
$= F \equiv (F \vee F)$
$= F \equiv F$
$= T$

After studying the various truth functions, a student may get an impression that all uses of words like "and", "either or", etc. make truth functionally compound propositions. But this is not so. There are non-truth functional compound propositions also. The following propositions, for example, are not truth functionally compound:

Radha and Rama are sisters
Mohan and Gopal are friends
Sita and Gita are twins
He believes that he is right.

A compound statement or a compound proposition is truth functionally compound if and only if its (compound proposition's) truth value depends on the truth value of its components or constituents. In other words, a truth functional statement is that statement whose truth value depends on words like "not", "and", "either" or "if then", etc.

Interdefinability of Truth Functions (Constants)

Some of the truth functions are basic while some others have merely theoretical value. Basic truth functions are ., ~, ⊃, v whereas ≡, ∧, / (stroke) are mainly for the theoretical purposes. Each truth function has its own special characteristics yet they can be reduced and defined into each other. A conjunction, for example, can be defined into other functions, such as "~" and "v", "~" and "⊃". Disjunction can similarly be defined into other

functions. **Defining and reducing the truth functions or constants into one another is called "interdefinability of the constants".** In other words, function like ~, •, v, ⊃ ≡ are dispensable signs. This can be shown by defining one function into other functions. However, while defining one truth function (or constant) into other truth functions (constants), the meaning of the expression should not be changed or destroyed.

Conjunctive function can be defined into:

(i) "~" and "v"
"(p•q)" = df "~ (~ p v ~ q)"

(ii) "~" and "⊃" as
"(p•q)" = df "~ (p ⊃ ~ q)"

The truth value of (p•q) is same as of (~ p v ~ q) or ~ (p ⊃ ~ q).

Ram is intelligent and Ram is hardworking can be defined in two ways:

1. It is false that either he is not intelligent or he is not hard-working.
2. It is false that if he is intelligent then he is not hardworking.

Disjunctive function can be defined into:

(i) "~" and "•'
"(pvq)" = df "~(~p•~q)"

(ii) "~" and "⊃'
"(pvq)" = df "~ p ⊃ q"

The truth value of pvq is same as that of ~(~p•~q) or (~p ⊃ q).

Either the opposition demands are met or they will boycott President's address, can be defined in two ways.

(1) It is false that neither opposition demands are met nor they boycott the President's address.

(2) If the opposition demands are not met, then they boycott the President's address.

Implicative function can be defined into:

(i) "~" and "v"
"(p ⊃ q)" = df "(~pvq)"

(ii) "~" and "•"
"(p ⊃ q)" = df "~ (p• ~q)"

If it rains, then the harvest will be good can be defined in two ways:

(1) Either it does not rain or the harvest will be good.

(2) It is false that it rains and the harvest is not good.

Equivalent function can be defined into:

(i) "." and "⊃'
"(p ≡ q)" = df "(p⊃q)"• (q ⊃ p)"

(ii) "~" and "•" and "v'
"p≡q" =df "(p•q) v (~p• ~q)"

The opposition attends the session if and only if their demand of inquiry is accepted can be expressed in two ways:

(1) If the opposition attends the session then their demand for inquiry is accepted, and if their demand for inquiry is accepted then they will attend the session.

(2) Either the opposition attends the session and their demand for inquiry is accepted or neither the opposition attends the session nor their demand for inquiry is accepted.

Alternative function can be defined into:

(i) "p∧q" = df "(pvq)• ~(p•q)"

(ii) "p∧q" = df "(pvq) • (~pv~q)"

Both equivalent and alternative functions are not basic functions, but they can be defined in terms of basic functions.

Stroke Function

Five logical constants (truth functions) • v ⊃ ≡ ∧ are defined into one and another with the help of "~". The question arises how can "~" as a constant be defined? The interdefinability of

truth functions is possible only with the help of negation. But none of the constants • ⊃ v ≡ helps in defining "not". **Thus in order to define negation, another new logical function was introduced by Sheffer in 1913. It is called stroke function.** The primary aim of the stroke function was to define negative function. Later on all other logical functions were also reduced into this single function of stroke. Stroke is symbolized as "/ " and is true if and only if at least one of its components is false.

p	q	p/q
T	T	F
T	F	T
F	T	T
F	F	T

The table shows that if both p and q are true, then p/q is false and if either of the variable is false, then p/q is true.

Let us see how other functions are defined in terms of stroke function.

1. ~p is defined as p/p.
   "~p" = df "p/p".
2. p•q is defined as (p/q)/(p/q)
   "p•q" = df "(p/q)/(p/q)"
3. pvq is defined as (p/p)/(q/q)
   "pvq" = df "(p/p)/(q/q)"
4. p ⊃ q is defined as p/(q/q)
   "p ⊃ q" = df "p/(q/q)"

Exercise 12

(A) If A, B, C are true statements and X, Y, Z are false, determine the truth value of the following propositions:

1. (X ⊃ Z) ⊃ ~Y
2. (A ⊃ ~ B) v ~ C
3. X v (~ Z • ~ A)
4. (B v X) ⊃ (B • Y)
5. (A v B) • (Z • C)
6. (C • Y) v (Z v B)
7. (~ B v ~ X) ⊃ (B v X)
8. (C • B) ⊃ (~ Z • ~ X)
9. ~ (A • B) v ~ (X v Y)
10. ~ (C ⊃ ~ B) ⊃ ~ (~ X ⊃ ~ Y)
11. ~ (Z V A) ⊃ (~ X • ~ A)
12. (A • B) ⊃ (~ Z • ~ X)
13. (X ⊃ Y) ⊃ (~ Z ≡ ~ A)
14. [(C • X) ⊃ Y] ⊃ (~A ⊃ ~Y)
15. [(A ⊃ B) v C] ≡ (X ⊃ A)
16. [B ⊃ (X • Y)] ⊃ [(X • Z) v B]
17. [(C • X) ⊃ C] ⊃ [(A ⊃ X) ⊃ C]
18. [(A • B) • C] ⊃ [(A v X) v Y]
19. [(A ⊃ C) ⊃ Y] ⊃ ~ (X • ~ Y)
20. [(~B • Y) ⊃ B)] ⊃ [A ⊃ (B ⊃ X)]
21. [B ⊃ (A v X)] ⊃ [(C ⊃ A) ⊃ Y]
22. [{(A ⊃ B) • X} ⊃ A] ⊃ ~ (B ⊃ X)
23. ~[(~C v Z) v (~Z v B)]
24. ~ [~ (A ⊃ B) v ~ (~ X v ~Y)]
25. ~ [(B ⊃ ~A) v ~ C] ⊃ (A ≡ B)
26. [(A v Y) • ~B] v [(A • B) v (Z • C)]

27. $[\sim B \supset (Y \bullet Z)] \supset \sim [(A \bullet X) \supset (C \bullet X)]$
28. $[(A \vee X) \supset C] \bullet [(X \bullet Z) \vee (A \vee Y)]$
29. $[(X \bullet Y) \supset B] \vee [(Y \bullet Z) \vee (A \bullet Y)]$
30. $[A \supset (X \bullet Y)] \bullet [(A \supset X) \vee (A \supset Y)]$
31. $[(C \bullet X) \supset Y] \equiv [(B \supset Y) \bullet (C \supset Z)]$
32. $[(B \bullet X) \vee (\sim A \bullet \sim X)] \bullet [(A \supset X) \bullet (X \supset A]$
33. $\sim [(\sim A \bullet \sim X) \vee A] \vee [(A \supset \sim X) \bullet (\sim B \supset \sim Y)]$
34. $[(A \supset C) \supset (X \bullet \sim Y)] \supset \sim [(- X \vee Y) \supset (A \vee X)]$
35. $[(B \supset B) \supset X] \supset [(X \supset \sim Y) \supset Z]$
36. $\sim [(C \supset B) \bullet (X \supset Y)] \equiv [(A \vee X) \supset (B \supset Y)]$
37. $\sim [(X \supset A) \bullet (Y \vee B)] \vee \sim [(X \vee A) \bullet (Y \vee B)]$
38. $\sim [(B \bullet X) \vee (\sim A \bullet \sim X)] \supset \sim [(A \supset X) \equiv (X \supset A)]$
39. $[\sim (C \supset B) \supset \{(A \supset B) \vee C\}] \equiv [(X \supset Y) \supset \{(X \supset Y) \vee Z\}]$
40. $\sim[\{(A \bullet X) \supset (B \supset Y)\} \vee \sim \{X \supset (Y \supset Z)\}]$

(B) If A and B are true and X and Y are false, but the values of P and Q are unknown, then what can be said about the truth value of the following propositions:

1. $(P \supset X) \supset (\sim X \supset \sim P)$
2. $[(A \supset X) \vee (Y \supset P)] \equiv [(P \bullet \sim P) \bullet Q]$
3. $[(P \supset X) \bullet (X \supset Q)] \supset [(P \bullet Q) \vee \sim (P \bullet Q)]$
4. $[Q \vee (B \bullet Y)] \supset [(Q \vee B) \bullet (Q \vee Y)]$
5. $\sim [P \supset (P \supset Q)] \supset (X \supset \sim A)$

(C) Define the following expressions into stroke function:

1. $\sim p \bullet q$
2. $p \vee \sim q$
3. $\sim (p \supset q)$
4. $(p \supset q) \vee r$
5. $p \vee (q \bullet r)$
6. $(p \supset q) \bullet r$

7. $(p \supset q) \supset r$
8. $(p \vee r) \supset q$
9. $p \supset (q \supset p)$
10. $\sim p \supset (q \bullet r)$
11. $p \supset (p \bullet \sim q)$
12. $(\sim p \vee q) \bullet q$
13. $(p \supset q) \vee \sim p$
14. $(p \bullet q) \supset (p \vee q)$
15. $\sim (p \bullet q) \bullet r$
16. $\sim p \vee (p \supset \sim q)$
17. $(\sim p \bullet \sim q) \supset r$
18. $p \equiv \sim q$
19. $\sim (p \bullet q) \bullet (q \supset p)$
20. $(p \bullet \sim p) \vee (p \bullet \sim q)$

(D) Define the following expressions into "~" and "v".

1. $(\sim p \supset q)$
2. $(p \bullet q) \vee r$
3. $p \bullet (\sim q \vee p)$
4. $(p \supset q) \supset p$
5. $\sim p \supset (\sim q \bullet r)$
6. $(\sim p \supset \sim q) \vee r$
7. $(q \bullet r) \supset p$
8. $p \supset (q \supset r)$
9. $(p \bullet q) \supset (p \vee q)$
10. $\sim (p \bullet q) \vee (q \supset p)$

(E) Define the following expressions into "~" and "•".

1. $\sim p \vee q$
2. $p \supset \sim q$

3. $\sim (p \vee q)$
4. $(p \bullet q) \vee r$
5. $p \vee (\sim q \supset p)$
6. $(p \supset q) \supset p$
7. $\sim p \supset (\sim q \vee r)$
8. $(p \vee r) \supset q$
9. $(\sim p \supset \sim q) \vee (r \bullet p)$
10. $p \equiv q$

Chapter 12

Truth Table Method as Decision Procedure

As we have seen in the last two chapters, the aim of symbolic logic is twofold — firstly to symbolize the arguments and expressions, and secondly to evaluate them. This chapter deals with the latter objective. In the logic of propositions there are many methods to test the validity of arguments and argument forms, and also to examine whether statements or statement forms are tautologous or not. These methods are called **decision procedures,** for they help in taking a decision whether an argument is valid or invalid, and also whether statement is tautologous or not. Some standard decision procedures in the logic of propositions are:

(1) Truth table method

(2) Shorter truth table method (*reductio ad absurdum*)

(3) CNF (Conjunctive Normal Form)

(4) DNF (Disjunctive Normal Form)

(5) Truth tree method.

The present chapter, however, deals with the truth table method only.

Truth Table Method

Truth table method is the most basic and perhaps the most popular method to examine the validity of arguments as well as logical status of statements. Let us see how the method is applied. First of all the number of simple propositions in the argument are counted. If a simple proposition is repeated in the argument or in the expression, then it is counted only once. In the expression

p ⊃ (p v q), the number of propositional variables are 2, since p occurs twice, it will be considered only once. In the expression ~q v [q v (r.p)], the number of propositional variables are 3, since q is repeated, it will be considered only once.

In the following argument form:

p ⊃ q

q ⊃ r

~s • ~ p ∴ p

the propositional variables are 4.

After knowing the number of propositional variables, the number of rows are decided. This depends on the number of propositional variables. If the number of propositional variables is 3, the number of rows are $2^3 = 2 \times 2 \times 2 = 8$ and if the propositional variables are 4, then the number of rows are $2^4 = 2 \times 2 \times 2 \times 2 = 16$.

The base is always 2 because we are dealing with two valued logic only. A proposition has two values, true or false. The power of 2 or the exponent of 2 is according to the number of propositional variables an argument form or an expression has. In the expression ~ p v [q v (r v p)] since the number of propositional variables are 3, the number of rows are $2^3 = 2 \times 2 \times 2 = 8$.

(p ⊃ q) • (r ⊃ t) has 4 variables, so the rows are $2^4 = 2 \times 2 \times 2 \times 2 = 16$. In the argument form:

p ⊃ q

r ⊃ s

s • t / ∴(q ⊃ r)

since the propositional variables are 5, the number of rows are $2^5 = 32$.

After counting the number of variables and deciding the number of rows, the truth table is constructed. Here some examples are given which show how the truth table is constructed.

(1)

(p	⊃ q)	v	p
T	T		T
T	F		T
F	T		F
F	F		F
1	2		3

In the above statement form, there are two propositional variables and hence total number of rows are $2^2 = 2 \times 2 = 4$. Under the first propositional variable two Ts, two Fs occur, total 4 times. Under the second variable one T and one F occurs, total 4 times. Since p is repeated, the same value of p (two Ts and two Fs) are substituted.

(2) [(p ⊃ q) • (r ⊃ p)] ⊃ q

Here, the number of propositional variables are 3, so number of rows are $2^3 = 2 \times 2 \times 2 = 8$. Under first variable four Ts, four Fs occur total 8 times. Under the second variable, two Ts, two Fs occur 8 times, and under the third variable one T and one F total 8 times.

The rule is: under the first variable half of the total number of rows are Ts and half are Fs. Under the second variable half of the first variable Ts and half of the first variable Fs are assigned. Under the third variable half of the second variable's Ts and half of the second variable's Fs are assigned.

[(p	⊃	q)	•	(r	⊃	p)]	⊃	q
T		T		T		T		T
T		T		F		T		T
T		F		T		T		F
T		F		F		T		F
F		T		T		F		T
F		T		F		F		T
F		F		T		F		F
F		F		F		F		F

(3) (~p . p) v p

Number of propositional variable = 1

Number of rows are $2^1 = 2$

(~p •	p)	v p
T	T	T
F	F	F

(4) ~(~ p • ~ q) ≡ (p v q)

Number of propositional variables = 2

Number of rows = $2^2 = 2 \times 2 = 4$

~ p •	~ q)	≡	(p	v q)
T	T		T	T
T	F		T	F
F	T		F	T
F	F		F	F

(5) {(p v q) • (q ⊃ r)} ⊃ (p ⊃ r)

Number of propositional variables are 3

Number of rows = $2^3 = 2 \times 2 \times 2 = 8$

{(p v q) • (q ⊃ r)} ⊃ (p ⊃ r)

T	T	T	T	T	T
T	T	T	F	T	F
T	F	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	T	T	F	F	F
F	F	F	T	F	T
F	F	F	F	F	F

After constructing truth table, the variables are joined using the properties of constants. First brackets are solved. Let us see some solved examples.

Example 1:

(p ⊃ q) v p

Number of propositional variables = 2

Number of rows = $2^2 = 2 \times 2 = 4$

(p	⊃	q)	v	p
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	T	F	T	F
1	3	2	5	4

Just as in mathematical expressions, here also first the brackets are considered. In this example 'p ⊃ q' is in bracket, so it is solved first. We know an implication (⊃) is false when the antecedent is true and consequent is false. After solving the bracket, the other part of the expression is solved. A disjunction is true when at least one of the disjuncts is true. The main column is under 5. **If under the main column there are all Ts then the expression form is *tautology*.** This statement form is tautologous for there are all Ts under the main column.

Example 2 :

(p v q) ⊃ (q • p)

Number of propositional variables = 2

Number of rows = $2^2 = 2 \times 2 = 4$

(p	v	q)	⊃	(q	•	p)
T	T	T	T	T	T	T
T	T	F	F	F	F	T
F	T	T	F	T	F	F
F	F	F	T	F	F	F
1	3	2	7	4	6	5

The main column (No. 7) contains some Ts and some Fs. Hence this statement form is *contingent*.

(3) (p ⊃ q) • ~ (p ⊃ q)

Number of prepositional variables = 2

Number of rows = $2^2 = 2 \times 2 = 4$

(p	⊃	q)	•	~	(p	⊃	q)
T	T	T	F	F	T	T	T
T	F	F	F	T	T	F	F
F	T	T	F	F	F	T	T
F	T	F	F	F	F	T	F
1	3	2	8	7	4	6	5

The main column (No. 8) contains all Fs, therefore, the above statement form is self-contradictory or contradictory.

A proposition, a propositional form, a statement, a statement form or an expression is either *tautology* or *contingent* or *self-contradictory*. A propositional form is tautology when under the main column there are all Ts; it is contingent when under the main column there are both Ts and Fs, and it is self-contradictory when under the main column there are all Fs. In other words, the logical status of a propositional form is either tautologous or contingent or self-contradictory.

Illustrated Statement forms

Test by truth table which of the following statement forms are tautologous, contingent or self-contradictory:

or

Examine the logical status of the following propositional forms.

(1)	(p	⊃	q)	v	(~	p	v	~	r)
	T	T	T	T	F	T	F	F	T
	T	T	T	T	F	T	T	T	F
	T	F	F	F	F	T	F	F	T
	T	F	F	T	F	T	T	T	F
	F	T	T	T	T	F	T	F	T
	F	T	T	T	T	F	T	T	F
	F	T	F	T	T	F	T	F	T
	F	T	F	T	T	F	T	T	F
	1	3	2	9	5	4	8	7	6

Since the main column (9) contains both Ts and Fs, the statement form is contingent.

(2)	(p	•	q)	⊃	[(p	v	q)	•	p]
	T	T	T	T	T	T	T	T	T
	T	F	F	T	T	T	F	T	T
	F	F	T	T	F	T	T	F	F
	F	F	F	T	F	F	F	F	F
	1	3	2	9	4	6	5	8	7

Since there are all Ts in the main column (9), therefore, the above statement form is tautologous.

(3)	(p	v	q)	•	(~	p	•	~	q)
	T	T	T	F	F	T	F	F	T
	T	T	F	F	F	T	F	T	F
	F	T	T	F	T	F	F	F	T
	F	F	F	F	T	F	T	T	F
	1	3	2	9	5	4	8	7	6

Since there are all Fs in the main column (9), the expression form is self-contradictory.

The truth table method can also be used to examine the bi-conditional of the propositional forms. Propositional forms are equivalent if their truth tables are same.

Illustrated examples:

(1)

(p	$\supset$	q)	$\equiv$	($\sim$	p	$\vee$	q)
T	T	T	T	F	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	F
1	3	2	8	5	4	7	6

Since the truth tables of both the expressions (column 3 and 7) are same, they are equivalent. The column number 8 contains all Ts which further confirms that both the expressions are bi-conditional.

(2)

(p	$\supset$	q)	$\equiv$	($\sim$	p	$\bullet$	$\sim$	q)
T	T	T	F	F	T	F	F	T
T	F	F	T	F	T	F	T	F
F	T	T	F	T	F	F	F	T
F	T	F	T	T	F	T	T	F
1	3	2	9	5	4	8	7	6

Since the truth tables of both the expressions are different (columns 3 and 8 have different truth tables), the above expressions are not bi-conditional.

(3)

(p	$\bullet$	q)	$\equiv$	(p	$\vee$	q)
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	F	T	F	F	T	T
F	F	F	T	F	F	F
1	3	2	7	4	6	5

Not bi-conditional.

Exercise 13

(A) Use truth table method to find out which of the following propositional forms are tautologous, contingent or self-contradictory.

1. $p \supset (p \supset q)$
2. $(p \bullet q) \supset r$
3. $p \supset (p \vee q)$
4. $(p \bullet p) \supset p$
5. $(p \vee p) \supset \sim p$
6. $p \supset (\sim p \supset \sim q)$
7. $p \supset (q \supset \sim q)$
8. $p \supset (q \supset \sim r)$
9. $(p \vee \sim p) \vee q$
10. $(p \bullet \sim p) \supset q$
11. $\sim (p \vee q) \vee \sim r$
12. $(\sim p \bullet \sim q) \bullet r$
13. $p \supset \sim (q \supset \sim r)$
14. $(p \supset q) \supset \sim (p \bullet r)$
15. $(\sim p \vee q) \supset (\sim p \bullet \sim p)$
16. $(p \supset q) \supset (\sim p \bullet \sim q)$
17. $(p \vee \sim p) \supset (p \bullet \sim p)$
18. $\sim (p \bullet q) \bullet (q \supset p)$
19. $(p \bullet \sim p) \vee (p \bullet \sim p)$
20. $[q \equiv (p \supset q)] \supset p$
21. $[(p \supset q) \bullet \sim p] \supset \sim q$
22. $[(p \supset q) \bullet p] \supset q$
23. $p \supset \sim [p \supset (p \bullet q)]$
24. $[(p \vee q) \supset r] \supset (p \supset r)$
25. $[\sim p \bullet (q \vee r)] \supset (q \vee r)$

26. $\sim [p \supset (q \bullet r)] \vee [(p \vee q) \supset r]$
27. $[(p \supset (q \supset r)] \supset [(p \vee q) \supset r]$
28. $(\sim p \supset \sim r) \supset \sim (\sim p \bullet \sim q)$
29. $p \supset [(p \supset q) \supset (\sim q \vee \sim q)]$
30. $[(p \supset q) \supset (q \sim r)] \supset [p \supset (q \bullet r)]$
31. $[(p \bullet q) \supset r] \vee [\sim p \supset (q \supset r)]$
32. $(p \vee q) \supset [(\sim p \supset r) \bullet (\sim q \supset r)]$
33. $(p \vee q) \supset [(\sim p \bullet r) \vee (\sim q \bullet r)]$
34. $[p \supset (q \supset r)] \supset [(p \bullet q) \supset r]$
35. $[(\sim p \supset \sim q) \supset \sim r] \supset \sim [p \supset (q \supset r)]$
36. $(p \supset q) \supset [(r \supset p) \supset (r \supset q)]$
37. $(p \vee q) \supset [(\sim p \supset r) \bullet (\sim q \bullet r)]$
38. $(p \vee q) \supset [(\sim p \bullet r) \vee (\sim q \vee r)]$
39. $[(p \supset q) \bullet (q \supset r)] \supset (\sim p \vee r)$
40. $(\sim p \equiv q) \equiv [(\sim p \supset q) \bullet (q \supset \sim p)]$
41. $[\{(p \vee q) \supset r\} \bullet \sim r] \supset \sim q$
42. $[p \bullet (q \vee r)] \supset [(p \bullet q) \vee (p \bullet r)]$
43. $(p \supset r) \supset [(p \bullet q) \vee (p \bullet r)]$
44. $[(p \supset \sim q) \vee r] \supset [(p \vee q) \supset r]$
45. $[(\sim p \supset \sim q) \supset \sim r] \supset [(q \supset r) \equiv (p \vee r)]$
46. $(p \bullet \sim q) \supset [(p \supset q) \supset \sim (p \bullet \sim q)]$
47. $[(p \bullet q) \supset r] \supset [(q \bullet \sim r) \supset \sim p]$
48. $[(p \supset \sim q) \supset (\sim p \supset q)] \supset \sim (p \supset r)$
49. $[(\sim p \supset q) \bullet (q \supset r) \bullet (\sim r \supset s)] \supset \sim (\sim p \supset s)$
50. $[(p \supset (\sim q \supset r) \bullet \{(p \bullet \sim r) \vee q\}] \supset (\sim q \bullet p)$

(B) Examine by truth table method which of the following are bi-conditional:

1. $(p \supset q) \equiv (\sim p \supset \sim q)$
2. $(p \bullet q) \equiv \sim (p \vee \sim q)$

3. $(\sim p \supset q) \equiv (\sim q \supset p)$
4. $(p \bullet q) \equiv \sim (p \vee q)$
5. $(p \bullet q) \equiv (\sim p \vee \sim q)$
6. $(\sim p \bullet \sim q) \equiv \sim (p \vee q)$
7. $(p \bullet q) \equiv \sim (\sim p \vee \sim q)$
8. $(p \bullet q) \equiv (\sim q \supset \sim p)$
9. $\sim (p \bullet q) \equiv (\sim p \bullet \sim q)$
10. $\sim (p \supset q) \equiv (p \vee q)$
11. $\sim (p \bullet \sim q) \equiv (q \vee \sim p)$
12. $\sim (p \vee q) \equiv (\sim p \bullet \sim q)$
13. $(\sim p \vee q) \equiv \sim (p \bullet q)$
14. $\sim (p \bullet q) \equiv (p \supset q)$
15. $[(p \bullet q) \vee r] \equiv q$
16. $p \equiv [(p \supset (p \bullet q)]$
17. $(p \supset q) \equiv [p \supset (p \supset q)]$
18. $[\sim (p \vee q) \supset r] \equiv \sim (p \vee q)$
19. $\sim (p \vee q) \equiv [(p \vee q) \supset p]$
20. $[(p \bullet (q \vee r)] \equiv \sim r$
21. $p \equiv \sim [(p \supset \sim (p \bullet q)]$
22. $[(p \vee (p \bullet q)] \equiv [(p \bullet (p \vee q)]$
23. $[(p \supset q) \bullet p] \equiv q$
24. $[\sim (p \vee q) \supset p] \equiv [(p \bullet q) \vee \sim p]$
25. $[(p \vee q) \supset r] \equiv [(p \bullet q) \supset r]$
26. $[(p \bullet q) \bullet r] \equiv [p \vee (q \vee r)]$
27. $[(p \supset q) \supset r)] \equiv [p \supset (q \supset r)]$
28. $[(p \vee q) \vee r] \equiv [r \supset (p \supset q)]$
29. $[(p \supset q) \supset r] \equiv \sim [(p \bullet q) \vee r)]$
30. $[(p \vee (q \bullet r)] \equiv [(p \bullet q) \vee (p \bullet r)]$
31. $[(p \bullet q) \supset r] \equiv [p \supset (q \supset r)]$

32. (p ≡ q) ≡ [(p • q) v (~ p • ~ q)]
33. [p • (q v r)] ≡ [(p • q) v (p • r)]
34. (p ≡ q) ≡ [(p ⊃ q) • (q ⊃ p)]
35. [(p ⊃ q) ⊃ r] ≡ [~ r ⊃ ~ (~ p v q)]

Testing the Validity/Invalidity of the Argument Forms and Arguments by Truth Table Method

Whereas a statement form is either tautologous, contingent or self-contradictory, an argument form is evaluated as valid or invalid. In a valid argument form all the premisses jointly imply the conclusion. A valid argument form cannot have all true premisses if the conclusion is false. We can test the validity of arguments and argument forms by truth table method. The procedure of constructing truth table is the same and the logical constants have the same properties.

All the premisses of the argument form are combined by conjunction and conclusion is replaced by '⊃' sign. Look at the following illustrated argument forms:

Example 1:

(1) p ⊃ q

p / ∴ q

is rewritten as

[(p ⊃ q) • p] ⊃ q

after combining premisses and conclusion.

In this argument form there are two premisses 'p ⊃ q'and 'p'. They together imply the conclusion 'q'.

[(p	⊃	q)	•	p]	⊃	q
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F
1	3	2	5	4	7	6

Since the main column (number 7) contains all Ts, hence the above argument form is valid.

Example 2:

p ⊃ q

q ⊃ r / ∴ p

[(p	⊃	q)	•	(q	⊃	r)]	⊃	P
T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	T
T	F	F	F	F	T	T	T	T
T	F	F	F	F	T	F	T	T
F	T	T	T	T	T	T	F	F
F	T	T	F	T	F	F	T	F
F	T	F	T	F	T	T	F	F
F	T	F	T	F	T	F	F	F
1	3	2	7	4	6	5	9	8

Since the main column (number 9) contains Fs, the above argument form is invalid. One F in the main column makes an argument form invalid.

Example 3:

p • q / ∴ ~ p

There is only one premiss and that premiss does not imply the conclusion. Let us see how:

(p	•	q)	⊃	~	P
T	T	T	F	F	T
T	F	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F
1	3	2	6	5	4

Invalid argument form.

Example 4:

(p ⊃ q) / ∴ ~ (~ q ⊃ ~ p)

(p	⊃	q)	⊃	~	(~	q	⊃	~	p)
T	T	T	F	F	F	T	T	F	T
T	F	F	T	T	T	F	F	F	T
F	T	T	F	F	F	T	T	T	F
F	T	F	F	F	T	F	T	T	F
1	3	2	10	9	5	4	8	7	6

Since the main column contains Fs, the above argument form is invalid.

Example 5:

~ (p ⊃ q)

p / ∴ q

[~	(p	⊃	q)	•	p)]	⊃	q
F	T	T	T	F	T	T	T
T	T	F	F	T	T	F	F
F	F	T	T	F	F	T	T
F	F	T	F	F	F	T	F
4	1	3	2	6	5	8	7

Invalid argument form.

Example 6:

p v q

~p / ∴ q

[p	v	q)	•	~	p]	⊃	q
T	T	T	F	F	T	T	T
T	T	F	F	F	T	T	F
F	T	T	T	T	F	T	T
F	F	F	F	T	F	T	F
1	3	2	6	5	4	8	7

Valid argument form.

Example 7:

p ⊃ q

q ⊃ r / ∴ p ⊃ r

[(p	⊃	q)	•	(q	⊃	r)]	⊃	(p	⊃	r)
T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	T	F	F
T	F	F	F	F	T	T	T	T	T	T
T	F	F	F	F	T	F	T	T	F	F
F	T	T	T	T	T	T	T	F	T	T
F	T	T	F	T	F	F	T	F	T	F
F	T	F	T	F	T	T	T	F	T	T
F	T	F	T	F	T	F	T	F	T	F
1	3	2	7	4	6	5	11	8	10	9

Valid argument form.

Example 8: If A wins the first prize then either B wins the second prize or C is disappointed. B wins the second prize. Therefore, either C is disappointed or A wins the first prize.

A ⊃ (B v C)

B

∴ C v A

{[A	⊃	(B	v	C)]	•	B}	⊃	(c	v	A)
T	T	T	T	T	T	T	T	T	T	T
T	T	T	T	F	T	T	T	F	T	T
T	T	F	T	T	F	F	T	T	T	T
T	F	F	F	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T	T	T	F
F	T	T	T	F	T	T	F	F	F	F
F	T	F	T	T	F	F	T	T	T	F
F	T	F	F	F	F	F	T	F	F	F
1	5	2	4	3	7	6	11	8	10	9

Invalid argument.

Example 9: If it does not rain, then the harvest will be destroyed and inflation will increase. It is false that if it rains then inflation will increase. The harvest will be destroyed. Therefore, inflation will increase.

~ R ⊃ (D • I)

~(R ⊃ I)

D / ∴ I

[{~	R	⊃	(D	•	I)}	•	~	(R	⊃	I)	•	D]	⊃	I
F	T	T	T	T	T	F	F	T	T	T	F	T	T	T
F	T	T	T	F	F	T	T	T	F	F	T	T	F	F
F	T	T	F	F	T	F	F	T	T	T	F	F	T	T
F	T	T	F	F	F	T	T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	F	F	T	T	F	T	T	T
T	F	F	T	F	F	F	F	F	T	F	F	T	T	F
T	F	F	F	F	T	F	F	F	T	T	F	F	T	T
T	F	F	F	F	F	F	F	F	T	F	F	F	T	F
2	1	6	3	5	4	11	10	7	9	8	13	12	15	14

Invalid argument.

Example 10: If Mr. X is late, then he will miss the plane. If he misses the plane, then the captain will not be pleased. It is false that captain is pleased or Mr. X will not miss the plane. Mr. X will miss the plane. Therefore, Mr. X will be late.

Symbolization:

L ⊃ M

M ⊃ ~ P

~ (P v ~ M)

M

∴ L

[(L	⊃	M)	•	(M	⊃	~	P)	•	~	(P	v	~	M)	•	M]	⊃	L
T	T	T	F	T	F	F	T	F	F	T	T	F	T	F	T	T	T
T	T	T	T	T	T	T	F	T	T	F	F	F	T	T	T	T	T
T	F	F	F	F	T	F	T	F	F	T	T	T	F	F	F	T	T
T	F	F	F	F	T	T	F	F	F	F	T	T	F	F	F	T	T
F	T	T	F	T	F	F	T	F	F	T	T	F	T	F	T	T	F
F	T	T	T	T	T	T	F	T	T	F	F	F	T	T	T	F	F
F	T	F	T	F	T	F	T	F	F	T	T	T	F	F	F	T	F
F	T	F	T	F	T	T	F	F	F	F	T	T	F	F	F	T	F
1	3	2	8	4	7	6	5	14	13	9	12	11	10	16	15	18	17

Invalid argument.

Exercise 14

(A) Construct the truth table of the following argument forms and determine their validity/invalidity:

1. p • q
   p / ∴ r
2. p v q
   ~ r / ∴ p
3. p v q
   ~ p / ∴ ~ q
4. p ⊃ q
   ~ p v r / ∴ r
5. ~ (p ⊃ q)
   p v r / ∴ q v s
6. p ⊃ ~ q
   ~ q ⊃ r / ∴ r ⊃ ~ p
7. ~ (p ⊃ q)
   q ⊃ r / ∴ ~ r
8. ~ (p ⊃ q)
   q v r / ∴ ~ p ⊃ q
9. p ≡ ~ q / ∴ p

10. $p \supset (q \bullet r)$ $/ \therefore \sim(q \bullet r) \supset \sim p$
11. $\sim r \vee s$
$\sim p \supset s$ $/ \therefore r \supset p$
12. $p \supset q$
$\sim p \supset r$ $/ \therefore q \vee r$
13. $\sim p \supset q$
$\sim q$
$\sim p \vee r$ $/ \therefore p$
14. $(p \supset q) \bullet (p \supset r)$ $/ \therefore q \vee r$
15. $p \supset (q \supset r)$
$p \supset q$ $/ \therefore (p \supset r)$
16. $\sim(p \vee q)$
$p \supset q$ $/ \therefore p \bullet \sim q$
17. $(p \vee q) \supset (p \bullet q)$
$p \bullet q$ $/ \therefore p \vee q$
18. $p \vee (q \bullet \sim q)$
$p \supset \sim q$ $/ \therefore q \bullet \sim p$
19. $\sim p \supset q$
$\sim(p \supset r)$
$\sim r$ $/ \therefore p$
20. $p \supset (q \vee r)$
$p \supset \sim q$ $/ \therefore p \vee r$

(B) Symbolize the following arguments and determine their validity/invalidity by truth table method:

(1) If children do not eat protein, then they do not have powerful bones. But children have powerful bones. Therefore, they eat protein.

(2) Either Sachin is a good player or he is lucky. Playing in several test matches implies he is a good player. Hence, he is not lucky.

(3) If it rains, then Sita drinks tea. If it does not rain, then she takes cold drinks. Therefore, if Sita does not drink tea, she will take cold drinks.

(4) Either unicorns are fighters or dragons are fearful of fire. If dragons are not fearful of fire, then unicorn are not fighters. It is true that dragons are fearful of fire. Thus, unicorn are fighters.

(5) If the football team wins, then it does not boast its victory. If the football team does not win, then it is not ashamed of its defeat. It follows that football team neither boasts its victory nor is ashamed of its defeat.

(6) If Mohan is late, then he will miss the plane. If he misses the plane, then his boss will not be pleased. Either his boss is pleased or Mohan will not miss the plane. Therefore, Mohan will not be late.

(7) If peacock is bird and has feathers, then it is not mammal. A peacock is not bird if it does not have feathers. It is false that peacock is bird but does not have feathers. Thus, it implies that peacock is certainly not a mammal.

(8) The Congress party cannot win the election unless it brings down the prices of essential commodities. If the new economic policies are implemented, then the prices of essential commodities will come down. It follows either the Congress party does not win the election or the new economic policies are not implemented.

(9) Either the police chief does not know the truth or he is hiding something in the interest of country. If he is hiding something in the interest of country, then the Parliament should provide him necessary security. Therefore, it is not the case that if the police chief knows the truth, then the Parliament will not provide him the necessary security.

(10) If the demand stays constant and the prices are lowered then the business will increase. If the lowering of the prices implies business to increase then we can control the market. Demand will stay constant. Thus, we can control the market.

Chapter 13

Shorter Truth Table Method
(Reductio ad absurdum or Indirect Method)

In the previous chapter you had noticed that the truth table method is an easy and simple decision procedure to test the validity of arguments, and to evaluate the expressions as tautologous. This is a mechanical[1] method and can be applied to a large number of arguments. But sometimes this method becomes very lengthy and it takes long to work it out. When the number of variables are considerably more, the number of rows subsequently increases and the truth table becomes quite big. In order to avoid this, there is another method which is shorter, and can be applied to all those cases where truth table is applicable. This new decision procedure is shorter truth table method or indirect method which is sometimes called *reductio ad absurdum* method also.

In this method we start by assuming that the argument form is invalid. By assuming it to be invalid, if we *do not meet any inconsistency* (while working the entire argument) then that argument form is actually *invalid*. But if by assuming it to be invalid we *do meet* any inconsistency then the argument form is proved valid. In other words, if the thesis is reduced to absurdity by assuming it to be wrong then indirectly it is valid. On the other hand, if the thesis is not reduced to absurdity by assuming

1. The truth table method is a mechanical procedure because there is one fixed *modus operandi* in applyng this method. Whenever the truth method is applied, its procedure is exactly the same. No new strategy has to be formed for evaluating the validity/invalidity of each argument or argument form. In this sense and only in this sense the method is mechanical.

it to be wrong, then indirectly it is invalid. The method of *reductio ad absurdum* consists in establishing the truth of a thesis by showing that the consequence of the truth of its contradictory is an inconsistency or self-contradictory.

Shorter truth table method is in a way backward counting of truth table method. In the truth table method one reaches to the main column in the end, but in the shorter truth table method one starts with the main column. The method can best be explained with the help of examples.

Example 1:

6	2	7	1	4	3	5
(p	$\supset$	q)	$\supset$	(q	$\supset$	p)
F	T	T	F	T	F	F

We begin by assuming the argument form to be invalid. It is assumed that there is at least one F in the main column. In the above example, the main column can have F only when the antecedent $(p \supset q)$ is true and consequent $(q \supset p)$ is false. We can proceed further either from the antecedent or from the consequent. But in the present case, it is better if we proceed with consequent $(q \supset p)$, for it is false only in one case (that is when antecedent is true and consequent is false), whereas $(p \supset q)$ is true in three cases. Further $(q \supset p)$ being false means q is true and p is false. We got the value of p as F and q as T. The value of these variables are assigned in $(p \supset q)$. After assigning these values in $(p \supset q)$ we find there is no inconsistency in the expression $(p \supset q) \supset (q \supset p)$. So at least one F under the main column is justified and the argument form is invalid.

Example 2:

$p \supset q$

$q \;/\therefore\; p$

As in the case of truth table method, in shorter truth table method also, all the premisses are combined with conjunction to form the expression like:

6	4	7	2	5	1	3
[(p	⊃	q)	•	q]	⊃	p
F	T	T	T	T	F	F

Invalid argument form.

We start by assuming argument form to be invalid. After assuming it to be invalid, we find no error or inconsistence in working out the entire expression, therefore, the argument form is invalid.

Example 3:

~ (q • p)

~ q / ∴ p

After joining the premisses and the conclusion the argument form is changed into the following expression:

4	8	7	9	2	5	6	1	3
[~	(q	•	p)	•	~	q]	⊃	p
T	F	F	F	T	T	F	F	F

Since there is no inconsistency, no error in assigning the values of variables, therefore, the argument form is invalid.

Example 4:

p ⊃ q

q ⊃ r /∴ p ⊃ r

10	6	11	2	9	7	8	1	4	3	5
[(p	⊃	q)	•	(q	⊃	r]	⊃	(p	⊃	r)
T	T	F	T	F	T	F	F	T	F	F
	F									
	12									

We started by assuming the entire argument form as invalid. From (p ⊃ r) we got the values of p and r as T, F respectively. By substituting the value of r in (q ⊃ r), we got q as F. By substituting the values of variables p, q, (p ⊃ q) becomes false

whereas it had to be true according to our assumption. How can the same expression (p ⊃ q) be true and false together? Thus, there is inconsistency, and this inconsistency is because of assuming F under the main column. The inconsistency can be removed if there is no F under the main column. Therefore, the argument form is valid.

Note: In (q ⊃ r), q can only be F, for 'q ⊃ r' is T and since value of r is F, q can only be F.

Indirect method or *reductio ad absurdum* method when applied to arguments or argument form is a complete decision procedure. It can evaluate an argument form as valid or invalid. One F under the main column justifies the invalidity of the argument form, and no F under the main column means argument form is valid.

Let us now apply the method to a statement form in order to evaluate whether they are tautologous or not. The procedure here is exactly the same as we have done in the case of argument forms.

Example 1:

7	9	8	2	10	12	11	1	3	5	4	6
{(p	⊃	q)	•	(q	⊃	r)}	•	~	(p	⊃	r)
T	T	T	F	T	F	F	F	T	T	F	F

Since there is no error, no inconsistency in assuming one F under the main column, therefore, the statement form is *not tautologous*.

But a non-tautologous expression can be either contingent or self-contradictory. How do, then, we know whether the above expression is contingent or self-contradictory? There is a way to do it. Let us see how it can be done. We will assume the expression has at least one T in the main column instead of F[2].

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2. The process is same whether we substitute T or F under the main column. By assuming either T or F if no inconsistency is met then the assumption (whether T or F) is correct.

9	7	10	2	11	8	12	1	3	5	4	6
{(p	⊃	q)	•	(q	⊃	r)}	•	~	(p	⊃	r)
T	T	T	T	T	T	F	T	T	T	F	F
					F						
					13						

By keeping one T in the main column there is inconsistency in working out (q ⊃ r). So the main column cannot have even one T. Hence, the above expression cannot be contingent; it is self-contradictory.

Example 2:

2	1	4	3	6	5	8	7
p	⊃	[p	⊃	(q	•	~	q)]
T	F	T	F	T	F	F	T

Since no inconsistency, no error, so the above expression is not tautology.

Now to prove whether it is contingent or self-contradictory, we will apply the method of *reductio ad absurdum* once again but instead of assuming F under the main column, we will assume one T.

2	1	4	3	6	5	7	8
p	⊃	[p	⊃	(q	•	~	q)]
F	T	F	T	T	F	F	T

By keeping one T under the main column no inconsistency is found, so the above expression has at least one T in the main column. The above expression thus is *contingent*.

Let us confirm this by truth table method:

1	8	2	7	3	6	5	4
p	⊃	[p	⊃	(q	•	~	q)]
T	F	T	F	T	F	F	T
T	F	T	F	F	F	T	F
F	T	F	T	T	F	F	T
F	T	F	T	F	F	T	F

The above expression is further confirmed contingent by applying truth table.

When an expression is proved non-tautologous by *reductio ad absurdum* method, then to determine whether that non-tautologous expression is contingent or self-contradictory, once again the *reductio ad absurdum* method is to be applied. But now instead of assuming the expression to be false, it is assumed to be true. By the same mechanism, if by substituting T in the main column, no inconsistency is met, then it means at least one T is justified in the main column, and hence the expression is contingent.

Exercise 15

(A) Examine by shorter truth table method which of the following expressions are tautologous:

1. (p ⊃ q) ⊃ (q v q)
2. (p • q) ⊃ (p ⊃ q)
3. (p v q) ⊃ (p ⊃ ~ q)
4. (p • q) ⊃ (~ q ⊃ r)
5. (p v q) ⊃ ~(p • q)
6. (p v q) v (~ p ⊃ ~ q)
7. (p • q) v (~ p ⊃ ~ q)
8. (p ≡ q) ⊃ (~ p v ~ q)
9. ~ (p • q) v ~ (p ⊃ q)
10. (p ⊃ q) ⊃ ~ (p • ~ q)

11. $(\sim p \supset q) \supset \sim(\sim q \supset \sim r)$
12. $\sim(p \vee \sim q) \supset \sim(p \bullet q)$
13. $(\sim p \supset \sim q) \supset \sim(p \bullet \sim q)$
14. $[(p \supset q) \bullet q] \supset p$
15. $[(p \vee q) \bullet (p \supset q)] \supset p$
16. $[\sim(p \bullet q) \bullet \sim p] \supset q$
17. $[(p \vee q) \supset r] \supset (p \vee q)$
18. $(p \supset q) \supset \sim[p \bullet (q \bullet r)]$
19. $[(p \bullet q) \supset \sim r] \supset (p \supset r)$
20. $(p \supset q) \supset [(\sim p \vee q) \vee r]$

(B) Examine by shorter truth table method which of the following expressions are tautologous:

1. $(p \supset q) \supset [p \supset (q \supset r)]$
2. $\sim[(p \vee q) \supset p] \supset q$
3. $(p \bullet q) \supset [q \supset (p \supset r)]$
4. $[p \supset (q \supset r)] \supset (p \supset r)$
5. $[(p \supset q) \supset r] \supset [p \supset (q \supset r)]$
6. $[(p \bullet q) \supset r] \supset [p \supset (q \supset r)]$
7. $[(p \supset q) \supset r] \supset [\sim p \supset (\sim q \supset \sim r)]$
8. $[p \supset (q \bullet r)] \supset [(p \supset q) \supset r]$
9. $[p \supset (q \bullet r)] \supset [p \supset (q \supset r)]$
10. $[p \supset (q \supset r)] \supset [(p \bullet q) \supset r]$
11. $[(q \supset p) \bullet \sim q] \supset (p \vee q)$
12. $[(p \supset q) \bullet (q \supset r)] \supset (p \supset r)$
13. $(p \supset q) \supset [(r \supset p) \supset (r \supset q)]$
14. $(p \supset q) \supset [(q \supset r) \supset (p \supset r)]$
15. $[(p \supset q) \supset r] \supset [(p \supset q) \supset (p \supset r)]$
16. $[p \supset (q \supset r)] \supset [(p \supset q) \supset (p \supset r)]$
17. $(p \supset q) \supset [(\sim p \supset r) \supset (p \supset (q \supset r))]$

18. $[(p \supset q) \bullet (r \supset s) \bullet (p \vee r)] \supset (q \supset s)$
19. $[(p \supset q) \bullet (p \supset r)] \supset [p \supset (q \supset r)]$
20. $[(p \supset q) \bullet (s \supset r) \bullet (q \supset s)] \supset (p \vee r)$

(C) Examine by *reductio ad absurdum* method the validity/invalidity of the following argument forms:

1.	$\sim(p \vee q)$	$/ \therefore \sim(p \bullet \sim q)$
2.	$p \supset q$ $p \vee q$	$/ \therefore \sim(p \bullet q)$
3.	$p \supset q$ $\sim r$	$/ \therefore q \supset \sim p$
4.	$p \supset q$ $\sim q$	$/ \therefore p \supset r$
5.	$p \vee q$	$/ \therefore \sim(\sim p \bullet \sim q)$
6.	$p \supset q$ $\sim q$	$/ \therefore \sim p \vee q$
7.	$p \supset q$ $\sim p$	$/ \therefore \sim q$
8.	$p \supset q$	$/ \therefore (q \bullet q) \supset \sim p$
9.	$p \supset (q \supset r)$	$/ \therefore (p \bullet q) \supset r$
10.	$p \supset q$ $q \supset r$	$/ \therefore \sim p \vee r$
11.	$(p \vee q) \supset r$ p	$/ \therefore r$
12.	$p \vee \sim q$ $p \bullet q$	$/ \therefore \sim(p \supset q)$
13.	$p \supset q$	$/ \therefore (\sim p \vee q) \vee r$
14.	$p \supset q$ $p \supset r$ p	$/ \therefore p \vee r$
15.	$\sim p \supset q$ $q \supset r$	$/ \therefore \sim r \supset \sim p$

16. $p \supset (q \supset r)$
    $p \supset q$ $\quad / \therefore p \supset r$
17. $p \supset (q \supset r)$
    $r \supset \sim p$ $\quad / \therefore r \supset \sim q$
18. $(\sim p \supset \sim q) \supset \sim r$ $\quad / \therefore p \supset (q \supset r)$
19. $(p \supset q) \supset r$
    $p \vee q$ $\quad / \therefore q \vee r$
20. $p \equiv q$
    $q \equiv (r \bullet p)$
    p $\quad / \therefore r \bullet p$

(D) Examine by shorter truth table method the validity/invalidity of the following argument forms:

1. $(p \supset q) \supset r$
   $\sim r$ $\quad / \therefore \sim p \vee q$
2. $(p \supset q)$
   $q \supset r$ $\quad / \therefore p \vee r$
3. $p \supset \sim q$
   $p \vee r$
   $r \equiv p$ $\quad / \therefore \sim (p \bullet q)$
4. $p \supset (q \supset r)$
   $p \supset q$ $\quad / \therefore p \supset r$
5. $p \supset (q \supset r)$
   $q \supset (p \supset r)$ $\quad / \therefore (p \bullet q) \supset r$
6. $p \supset (q \supset r)$
   $r \supset \sim p$ $\quad / \therefore r \supset \sim q$
7. $p \supset q$
   $r \supset t$
   $p \vee r$ $\quad / \therefore q \vee t$
8. $q \supset (p \bullet r)$
   $r \supset \sim q$ $\quad / \therefore q \vee \sim r$

9. $p \supset (q \bullet \sim r)$
   $(q \vee r) \supset t$
   p $\quad / \therefore t$
10. $(p \vee q) \supset (p \bullet q)$
    $\sim p \supset \sim q$ $\quad / \therefore \sim (p \bullet q)$
11. $\sim p \vee q$
    $p \supset q$
    q $\quad / \therefore p$
12. $(p \supset q) \bullet (p \vee q)$
    $\sim p \vee \sim q$ $\quad / \therefore \sim (p \bullet q)$
13. $(p \supset q)$
    $\sim (p \supset r)$
    p $\quad / \therefore q \vee r$
14. $(p \bullet q) \bullet r$ $\quad / \therefore \sim (p \supset q) \supset r$
15. $(p \bullet q) \supset r$
    $\sim r$ $\quad / \therefore \sim (\sim p \vee \sim q)$

Chapter 14

Formal Proof of Validity

FORMAL proof of validity is a decision procedure. But it is a half decision procedure because if we are able to construct a formal proof, then that argument is valid. But if, on the other hand, we are not able to construct its formal proof then either the argument is invalid or we are not able to construct its formal proof. It is quite possible that the argument is valid but we are not able to form its formal validity. It is also possible that the argument may, in fact, be invalid and in that case how so ever hard we may try we can never construct its formal proof of validity. For the invalid arguments formal proof of validity can never be constructed. Thus there is a need for another decision procedure which can establish its validity or invalidity for sure.

Let us see how formal proof of validity is constructed. Constructing a formal proof of validity is like solving a puzzle. It is like a rat chasing cheese and it is a detective's hunt to solve mystery. In order to reach the target, one has to form a strategy. The detective too makes a planning to achieve the targeted aim.

Formal proof of validity is *not* a mechanical process. Unlike truth table method or any other decision procedure, the formal proof of validity does not have fixed *modus operandi*. Each argument needs different strategy. There is no master strategy which can solve all the mysteries. A swimmer negotiates each and every wave while swimming even though he is efficient swimmer. A driver also has to manipulate each turn while driving a vehicle. A good batsman assesses and examines each and every ball he faces. Each ball is challenge to him. Similarly in order to construct formal proof of validity one has to treat every argument differently.

There are nine Rules of Inference (and ten Rules of Replacement*) which help in constructing formal proof of validity. These rules are tautological and hence are valid forever. They are self-evident and thus are the basis of validity though they themselves are beyond proof. A set of nine rules serves like *tool kit* in constructing formal proof of validity.

In formal proof of validity some premisses are given and on that basis, conclusion is deduced with the help of these **Rules**. It resembles the geometrical process where in order to prove a certain theorem, the geometrician begins with certain given facts and axioms. He *starts* with some given statements and with the help of certain self-evident rules, he *deduces* the conclusion. This method is called *Deductive Method*. Thus within a defined system, a geometrical theorem is formally proved valid. Exactly in the same way in the formal proof of validity certain premisses are *given* and with the help of Rules of Inference conclusion is drawn from them.

The nine *Rules of Inference* are as follows:

1. Modus Ponens (M.P.)

$$\begin{array}{c} p \supset q \\ p \\ \hline \therefore q \end{array}$$

Modus Ponens means affirming the antecedent and on that basis affirming the consequent.

For example, the following two arguments are valid by this rule:

(i) If Ram is graduate, then he is eligible for the job.
Ram is graduate.
∴ He is eligible for the job.

* This book deals only with the Rules of Inference.

(ii) If I stay in the city, then I will have no peace of mind.
I stay in the city.
∴ I will have no peace of mind.

All the following are *valid* forms of Modus Ponens:

(i) $p \supset q$
$\underline{p}$
$\therefore q$

(ii) $\sim p \supset \sim q$
$\underline{\sim p}$
$\therefore \sim q$

(iii) $(p \bullet q) \supset r$
$\underline{p \bullet q}$
$\therefore r$

All the following are *invalid* forms of Modus Ponens:

(i) $\sim p \supset q$
$\underline{p}$
$\therefore q$

(ii) $p \supset q$
$\underline{\sim p}$
$\therefore \sim q$

2. Modus Tollens (M.T.)

$$\begin{array}{l} p \supset q \\ \underline{\sim q} \\ \therefore \sim p \end{array}$$

Modus Tollens means denying the consequent and hence denying the antecedent. For example, the following arguments are valid by Modus Tollens:

(i) If it rains, then we shall go for picnic.
We shall not go for picnic.
∴ It does not rain.

(ii) If there is storm tonight, then the nights will be cool.
The nights are not cool.
∴ There is no storm tonight.

(iii) If I win the case, then I will not have to pay penalty.
I pay the penalty.
∴ I will not win the case.

All the following are *valid* forms of Modus Tollens:

(i) $p \supset q$
$\underline{\sim q}$
$\therefore \sim p$

(ii) $p \supset \sim q$
$\underline{q}$
$\therefore \sim p$

(iii) $p \supset (q \bullet r)$
$\underline{\sim (q \bullet r)}$
$\therefore \sim p$

All the following are *invalid* forms of Modus Tollens:

(i) $\sim p \supset q$
$\underline{q}$
$\therefore \sim p$

(ii) $p \supset \sim q$
$\underline{\sim q}$
$\therefore \sim p$

3. Disjunctive Syllogism (D.S.)

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ \therefore q \end{array}$$

In Disjunctive Syllogism two options are given. If one option is denied then the other option is to be accepted. For example, the following arguments are valid because of Disjunctive Syllogism.

(i) Either the support is strong or we lose the election.
The support is not strong.

∴ We loose the election.

(ii) Either we play football or we eat hat dogs.
We will not play football.

∴ We eat hot dogs.

Yet another example:

(iii) Either he is fool or he is knave.
He is not fool.

∴ He is knave.

All the following are *valid* forms of disjunctive syllogism:

(i) $p \vee q$
$\sim p$
$\therefore q$

(ii) $\sim p \vee q$
p
$\therefore q$

(iii) $\sim p \vee \sim q$
p
$\therefore \sim q$

(iv) $(p \bullet q) \vee r$
$\sim (p \bullet q)$
$\therefore r$

All the following are *invalid* forms of Disjunctive Syllogism:

(i) $p \vee q$
p
$\therefore \sim q$

(ii) $\sim p \vee q$
$\sim p$
$\therefore \sim q$

(iii) $\sim p \vee q$
$\sim p$
$\therefore q$

(iv) $\sim p \vee \sim q$
$\sim p$
$\therefore q$

(v) $p \vee q$
q
$\therefore \sim p$

4. Hypothetical Syllogism (H.S.)

$$p \supset q$$
$$q \supset r$$
$$\therefore p \supset r$$

In this rule of inference both the premisses are conditional statements and they have one common component. The common component is antecedent in one premiss and consequent in another premiss. It can best be understood with the help of examples:

(i) If she studies humanities, then she prepares to earn a good living.
If she prepares to earn a good living, then her college days are well spent.

∴ If she studies humanities, then her college days are well spent.

(ii) If x is a politician, then x is interested in securing votes.
If x is interested in securing votes, then x must be attentive to the needs of the people.

∴ If x is a politician, then x must be attentive to the needs of the people.

The following are *valid* forms of Hypothetical Syllogism:

(i) $p \supset q$
$q \supset r$
$\therefore p \supset r$

(ii) $q \supset r$
$p \supset q$
$\therefore p \supset r$

(iii) $p \supset \sim q$
$\sim q \supset r$
$\therefore p \supset r$

(iv) $\sim p \supset \sim q$
$\sim q \supset \sim r$
$\therefore \sim p \supset \sim r$

The following are *invalid* forms of Hypothetical Syllogism:

(i) $p \supset q$
$r \supset q$
$\therefore p \supset r$

(ii) $p \supset q$
$p \supset r$
$\therefore q \supset r$

(iii) $p \supset q$
$p \supset r$
$\therefore r \supset q$

The rule states that in the Hypothetical Syllogism the common term should be antecedent in one premiss and consequent in the other premiss.

5. Constructive Dilemma (C.D.)

$$(p \supset q) \bullet (r \supset s)$$
$$p \vee r$$
$$\therefore q \vee s$$

Every dilemma has two conditional propositions (as its major premiss). In the Constructive Dilemma the major premiss is one having two different antecedents, both leading to the different consequent. Look at the following examples:

(i) If a man is rationalist, then he must forgo the enjoyment of many pleasures; and if he is a hedonist, he must often neglect his duty. A man is either a rationalist or a hedonist.

∴ Either he will forgo the enjoyment of many pleasures or he must often neglect his duty.

Look at another famous example of Constructive Dilemma:

(ii) If I do justice then God will love me and if I am unjust then politician will love me.

Either I do justice or I am unjust.

∴ Either God will love me or politician will love me.

All the following are *valid* forms of Constructive Dilemma:

(i) $(p \supset q) \bullet (r \supset s)$
$p \vee r$
$\therefore q \vee s$

(ii) $(\sim p \supset q) \bullet (\sim r \supset s)$
$\sim p \vee \sim r$
$\therefore q \vee s$

All the following are *invalid* forms of Constructive Dilemma:

(i) $(p \supset q) \bullet (r \supset s)$
$q \vee s$
$\therefore p \vee r$

(ii) $(p \supset q) \bullet (r \supset s)$
$p \vee s$
$\therefore q \vee r$

6. Conjunction (Conj.)

$$\frac{\begin{matrix} p \\ q \end{matrix}}{\therefore p \bullet q}$$

This rule states that all the premisses are joint together by using Conjunction. This device we had used earlier also in Truth Table Method and Shorter Truth Table Method. This rule can be expanded. See the following example:

$$\frac{\begin{matrix} p \\ q \\ r \end{matrix}}{\therefore p \bullet q \bullet r}$$

7. Simplification (Simp.)

$$\frac{p \bullet q}{\therefore p}$$

This rule is just the opposite of the earlier rule. If in the premiss Conjunction is the main sign, then the *first conjunction* can be inferred.

Following are *valid* forms of simplification:

(i) $\dfrac{p \bullet q}{\therefore p}$

(ii) $\dfrac{\sim p \bullet q}{\therefore \sim p}$

(iii) $\dfrac{(p \vee q) \bullet r}{\therefore p \vee q}$

Following are *invalid* forms of simplification:

(i) $\dfrac{(p \bullet q) \vee r}{\therefore p}$

(ii) $\dfrac{p \supset (q \bullet r)}{\therefore q}$

8. Addition (Add.)

$$\frac{p}{\therefore p \vee q}$$

This rule states one can add any variable or variables to the premiss with the help of wedge (v). The following are *valid* forms of Addition:

(i) $\dfrac{p}{\therefore p \vee q}$

(ii) $\dfrac{p}{p \vee (q \bullet r)}$

(iii) $\dfrac{p}{\therefore p \vee \sim p}$

(iv) $\dfrac{p}{\therefore p \vee (q \vee (p \bullet r))}$

The following are *invalid* forms of Addition:

(i) $\dfrac{p}{\therefore p \bullet (q \vee p)}$

(ii) $\dfrac{p}{\therefore p \supset (p \vee r)}$

9. Absorption (Abs.)

$$\frac{p \supset q}{\therefore p \supset (p \bullet q)}$$

In this rule, consequent carries (and absorbes) antecedent in the conclusion as stated above.

SOME SOLVED EXAMPLES

Construct Formal Proof of validity of the following argument forms and arguments:

(i)
1 $p \supset q$
2 $q \supset s$
3 p — $/ \therefore s$
4 $p \supset s$ — 1st and 2nd by H.S.
5 s — 4th and 3rd by M.P.

(ii)
1 $N \supset M$
2 $M \supset D$
3 $M \supset P$
4 $\sim P$
5 $N \vee M$ — $/ \therefore D$
6 $\sim M$ — 3rd and 4th by M.T.
7 $\sim N$ — 1st and 6th by M.T.
8 M — 5th and 7th by D.S.
9 D — 2nd and 8th by M.P.

(iii)
1 $(p \vee q) \supset r$
2 $s \vee p$
3 $\sim s$ — $/ \therefore r$
4 p — 2nd and 3rd by D.S.
5 $p \vee q$ — 4th by Add.
6 r — 1st and 5th by M.P.

(iv)
1 $p \supset q$
2 $r \supset s$
3 $p \vee r$
4 $(q \vee s) \supset t$ — $/ \therefore t$
5 $(p \supset q) \bullet (r \supset s)$ — 1st and 2nd by Conj.
6 $q \vee s$ — 5th and 3rd by C.D.
7 t — 4th and 6th by M.P.

(v)
1 $p \supset q$
2 $q \supset r$
3 $\sim p \supset (s \vee t)$
4 $\sim r$ — $/ \therefore s \vee t$
5 $p \supset r$ — 1st and 2nd by H.S.
6 $\sim p$ — 5th and 4th by M.T.
7 $s \vee t$ — 3rd and 6th by M.P.

(vi)
1 $\sim P \vee (I \vee E)$
2 $P \bullet R$
3 $\sim I$ — $/ \therefore E$
4 P — 2nd by Simp.
5 $I \vee E$ — 1st and 4th by D.S.
6 E — 5th and 3rd by D.S.

(vii)
1 $p \supset \sim q$
2 $\sim q \supset r$
3 $r \supset s$
4 $(p \bullet s) \supset u$ — $/ \therefore p \supset u$
5 $p \supset r$ — 1st and 2nd by H.S.
6 $p \supset s$ — 5th and 3rd by H.S.
7 $p \supset (p \bullet s)$ — 6th by Abs.
8 $p \supset u$ — 7th and 4th by H.S.

(viii)
1 $p \supset q$
2 $q \supset (s \vee r)$
3 $r \supset t$
4 $t \supset u$
5 $s \supset y$
6 p
7 $\sim y$ — $/ \therefore u$
8 $p \supset (s \vee r)$ — 1st and 2nd by H.S.
9 $s \vee r$ — 8th and 6th by M.P.
10 $\sim s$ — 5th and 7th by M.T.
11 r — 9th and 10th by D.S.
12 $r \supset u$ — 3rd and 4th by H.S.
13 u — 12th and 11th by M.P.

(ix) If it rains in time, there will be good crops. If there is good crops, then prices of essential commodities will not increase. It rains in time. Therefore the prices of essential commodities will not increase.

1 $R \supset C$
2 $C \supset \sim I$

3 R   / ∴ ~ I
4 C   1st and 3rd by M.P.
5 ~ I   2nd and 4th by M.P.

(x) If the students protest hike in fees then either the college will withdraw extra facilities or the government will have to spend more. The students will protest hike in fees. The college will not withdraw extra facilities. Therefore the government will have to spend more.

1 S ⊃ (F v G)
2 S
3 ~ F   / ∴ G
4 F v G   1st and 2nd by M.P.
5 G   4th and 3rd by D.S.

Exercise 16

A. Construct Formal Proof of Validity for each of the following argument forms:

1. p • ~ p   / ∴ p
2. p   / ∴ p v q
3. ~ p ⊃ q
   ~ p   / ∴ q
4. p ⊃ q
   p   / ∴ q v s
5. (p ⊃ q) ⊃ r
   ~ r   / ∴ ~ (p ⊃ q)
6. p ⊃ q
   q ⊃ ~ r   / ∴ p ⊃ ~ r
7. ~ J ⊃ K
   ~ J   / ∴ K
8. (~ S ⊃ H) ⊃ (~ H ⊃ S)
   (~ S ⊃ H)   / ∴ ~ H ⊃ S

9. p ⊃ q
   p • q   / ∴ q
10. p ⊃ (q ≡ r)
    (q ≡ r) ⊃ s   / ∴ p ⊃ s
11. ~ S ⊃ T
    T ⊃ ~ S   / ∴ ~ S ⊃ ~ S
12. (~ p ⊃ ~ q) • (r ⊃ s)
    ~ p v r   / ∴ ~ q v s
13. p ⊃ q
    q ⊃ r
    ~ r   / ∴ ~ p
14. p ⊃ (q v r)
    s ⊃ p
    s   / ∴ q v r
15. p ⊃ ~ q
    ~ q ⊃ r   / ∴ p ⊃ (p • r)

B. Construct Formal Proof of Validity for the following:

1. p ⊃ q
   q ⊃ s
   p   / ∴ s
2. (p v q) ⊃ r
   p   / ∴ r
3. p ⊃ q
   q ⊃ r
   r ⊃ s   / ∴ p ⊃ s
4. S ⊃ H
   H ⊃ ~ I
   I   / ∴ ~ S
5. p ⊃ (q • r)
   r ⊃ ~ (q • r)
   r   / ∴ ~ p

6. $q \vee p$
$r \vee \sim p$
$\sim q$
$\sim r$ $\quad / \therefore p \bullet \sim p$

7. $p \vee \sim q$
$r \supset q$
$\sim r \supset s$
$\sim p$ $\quad / \therefore s$

8. $A \supset B$
$B \supset C$
$\sim C \vee D$
A $\quad / \therefore D$

9. $S \supset W$
$W \supset H$
$D \vee \sim H$
$\sim D$ $\quad / \therefore \sim S$

10. $S \supset E$
$T \vee S$
$\sim T$ $\quad / \therefore E$

11. $R \supset W$
$W \supset D$
$\sim D$ $\quad / \therefore \sim R$

12. $\sim F \supset \sim M$
$M \vee P$
$\sim F$ $\quad / \therefore P$

13. $p \supset (q \vee r)$
p
$\sim q$ $\quad / \therefore r$

14. $p \vee q$
$\sim p$
$r \supset \sim q$ $\quad / \therefore \sim r \vee t$

15. $p \supset (q \supset r)$
$p \bullet t$
$r \supset s$ $\quad / \therefore q \supset s$

C. Construct Formal Proof of Validity of the following argument forms:

1. $p \supset q$
$q \supset r$
$r \supset s$
$s \supset t$ $\quad / \therefore p \supset t$

2. $\sim p \supset q$
$\sim p$
$\sim r \supset \sim q$ $\quad / \therefore \sim \sim r$

3. $p \supset (q \supset r)$
p
$\sim r$ $\quad / \therefore \sim q$

4. $p \supset (p \bullet q)$
$\sim (p \bullet q)$ $\quad / \therefore \sim p$

5. $(p \supset q) \bullet (r \supset s)$
$s \supset (p \vee r)$
s $\quad / \therefore q \vee s$

6. $p \supset q$
$q \supset r$
$r \supset s$
$p \vee q$
$\sim s$ $\quad / \therefore q \vee t$

7. $(p \bullet q) \supset \sim s$
$\sim s \supset t$
$r \vee \sim t$
$\sim r$ $\quad / \therefore \sim (p \bullet q)$

8. $A \vee (B \bullet C)$
$C \supset \sim B$
$\sim A$ $\quad / \therefore \sim C \vee D$

9. $(A \supset B) \bullet (C \supset D)$
$A \vee C$
$\sim B$ $\quad / \therefore D$

10. $p \supset (q \bullet r)$
$q \supset (s \supset p)$
$p \qquad / \therefore s \supset (s \bullet p)$

11. $C \vee [B \supset (A \supset D)]$
$\sim C$
$B \qquad / \therefore A \supset (A \bullet D)$

12. $(p \supset q) \bullet (s \supset t)$
$r \vee p$
$\sim r \qquad / \therefore q \vee t$

13. $[(p \vee q) \bullet (r \vee s)] \supset t$
p
$r \qquad / \therefore t$

14. $C \supset D$
$E \vee \sim D$
$\sim E \bullet A$
$(A \bullet \sim B) \supset C$
$(A \bullet \sim B) \vee B \quad / \therefore B$

15. $(F \bullet \sim T) \supset G$
$U \supset F$
$B \supset \sim T$
U
$B \qquad / \therefore G$

D. Construct Formal Proof of Validity of the following arguments:

1. If the market drops, Simon will borrow more money. Simon does not borrow more money. Therefore, the market does not drop (M, B).
2. If the witness is honest, he will speak truth. He is known to be hones. Therefore, he will speak truth (W, S).
3. If there is a drought, then grain will be dearer. Grains will not be dearer. Therefore, there is no drought (D, G).
4. If the support is not strong, then we do not win the election. The support is not strong. Therefore, we do not win the election (S, E).

5. If we are aggressive, then we are accused of hostile acts and if we are passive, then we are threatened by disaster. Either we are aggressive or we are passive. Therefore, either we are accused of hostile acts or we are threatened by disaster (A, H, P, T).
6. If a man goes to university, then he has a chance to develop his natural intelligence. If a man has inquisitive mind, then he will go to university. He never got a chance to develop his natural intelligence. Therefore, either he does not have inquisitive mind or he was not lucky (G, D, I, L).
7. If a manager makes an important decision, and if he wants to implement it, then he must be courageous. The manager makes an important decision. It is false that the manager is courageous. Therefore, the manager will not implement his decision (M, I, C).
8. A doctor cannot diagnose the disease, unless he has experience in the tropics. If the doctor has experience in the tropics, then the patient will not be very ill. The doctor can diagnose the disease. Therefore, the patient will not be very ill (D, T, L).
9. If the minority is not willing to accept the majority decision, there can be no democracy. If the majority does not respect the rights of the minority, then minority will not be willing to accept the majority decision. The majority does not respect the rights of the minority. Therefore, there cannot be democracy (W, D, R).
10. If taxes rise, then the cost of living rises, and if inflation continues, then wages are permitted to rise. Either the taxes rise or the inflation continues. The cost of living will not rise. If wages are permitted to rise, then the public will not be agitated. Therefore, neither the cost of living will rise nor the public will be agitated (T, L, I, W, A).

Chapter 15 — Section A

Predicate Calculus

George Boole in the mid-nineteenth century was the first logician to start calculus in logic. In the *Mathematical Analysis of Logic* and *An Investigation of the Laws of Thought*, he has given the precise idea of logical calculus. His logic of algebra can be divided into two parts: the calculus of classes and the calculus of proposition. So far what you have read in the previous few chapters of this book was a part of propositional calculus.

In propositional calculus proposition is taken as basic unit. In this branch of knowledge, subject matter of a proposition is not considered. The logicians are concerned only with the essential features common to all the propositions namely truth and falsehood. These twin values are usually called truth-values. In the propositional calculus, logicians focus on the various combinations of simple propositions. Since atomic or simple proposition is the smallest unit, they can be combined in various ways. This process makes truth functionally compound propositions such as "$p \bullet q$", "$p \vee q$", "$p \supset q$", "$p \equiv q$", etc. possible about which you have already read in the previous chapters.

There are, however, certain limitations of the propositional calculus. While symbolizing the propositions, their qualities (affirmation or negation) are shown whereas the quantities (universality or particularity) of the propositions are not made explicit. For example, all the following propositions.

1a. All voters are citizens.

1b. Some voters are citizens.

1c. Ram being a voter is citizen.

are symbolized as C as if they are same. But we all know they are different. Whereas (1a) is A proposition, (1b) is I proposition and (1c) is singular proposition.

Take another example. Both of the following propositions

2a. All men are wise
and
2b. Some men are wise

are symbolized as M. But the above two propositions are different because whereas (2a) is universal affirmative A proposition, (2b) is particular affirmative I proposition.

Similarly,

3a. No man is perfect
and
3b. Some men are not perfect

are treated symbolized as $\sim P$.

But the above two propositions are different because the former proposition is E and the latter is O.

Thus, in order to make this point clear another calculus was introduced which is called **Predicate Calculus,** sometimes it is referred as **Propositional Function.** This new calculus makes the "internal structure" of the propositions clear. Both qualities (affirmation and negation) and quantities (universality and particularity) of the propositions are made clear. Predicate calculus gives more information about the internal structure of the propositions.

In the predicate calculus, as the name suggests, weightage is given to the predicates of the propositions. The predicate may be assigned to the subject (affirmative propositions) or it may be denied to the subject (negative propositions). Also the predicate may be assigned (or denied) to all members such as in universal propositions; or to some members like particular propositions; or to a singular object like singular proposition.

It also further reveals that same predicate is assigned to two different subjects such as in the proposition "Ram and Mohan both are students of this college"; or different predicates are assigned to the same subject like in the proposition "Ram is intelligent and he is hard working." In predicate calculus, subject and predicate of a proposition are viewed as properties, characteristics or attributes. For example: "All men are mortal" is analysed as: If there is a thing which has the characteristics of a man, then it necessarily has the property of being mortal. The proposition "All leaves are green," suggests that the quality of green is a 'part' of the quality of leaves. This statement in the predicate calculus is explained as: "If a thing is leaf, then it has the property of being green." Here the **logicians are not concerned with characteristics but with the things that have characteristics.**

But at the same time it is wrong to think that predicate calculus is totally different from propositional calculus. Rather the predicate calculus takes many things from the propositional calculus; but it goes beyond Predicate calculus gives more information about the "internal structure" of the propositions. Basson and O' Connor states:

> We may thus regard the calculus of predicate as a branch of logic which includes not only the calculus of proposition but also goes beyond it. It includes it in the sense that, if any formula is valid in the calculus of proposition, the corresponding predicate formula will be valid in the predicate calculus. And goes beyond it in the sense that it makes the **structure** of its propositional material more explicit. And because of this it is able to deal with forms of argument which, on account of their complexity, are beyond the scope of the propositional calculus.[1]

The basic symbols frequently used in the predicate calculus are as follows:

1. 'x", 'y", 'z' are individual variables.

1. A.H. Basson, and D.J. O'Connor, *Introduction to Symbolic Logic*, p. 107.

2. "a", "b", "c" are individual constants for proper names.
3. "F", "G", "H" stand for predicates, characteristics and attributes.
4. The auxiliary for a universal proposition is small x within parentheses like it (x). It is read as "for all values of x"...
5. The auxiliary symbol for a particular proposition is (∃ x) and it is read as "there is at least one object x such that"...

Let us see how the categorical propositions are analysed and symbolized in the predicate calculus.

Universal affirmative A proposition "All cats are mammals" is analysed as:

⇒ (Whatever x may be) (If x is cat then it is mammal)
⇒ (x) (If x is C, then x is M)
⇒ $(x)\ (Cx \supset Mx)$

(x) stands for the universal quantity of the proposition. All universal propositions are symbolized by using the (x) or equivalent like (y), etc. which is known as Universal Quantifier, because it reveals universal quantity of the proposition.

E proposition "No child is voter" is analysed and symbolized as:

⇒ (Whatever x may be) (If x is a child then x is not a voter)
⇒ (x) (If x is a child, then x is not a voter)
⇒ (x) (If x is C, then x is ~V)
⇒ $(x)\ (Cx \supset \sim Vx)$

Turning now to particular propositions which are also called existential propositions, let us begin with 'I' proposition:

"Some students are Indian."

⇒ (There is something x) (This x is a student and this x is Indian)
⇒ (∃x) (This x is S and This x is I)
⇒ $(\exists x)\ (Sx \bullet Ix)$

'∃x' is read as "Ex" and is called existential quantifier. It represents particular propositions and in modern logic particular propositions are called existential propositions because they have existential import.

Proposition 'O' "Some students are not voters" is analysed and symbolized in the similar way such as:

⇒ (There is something x) (This x is a student and this x is not a voter)

⇒ (∃x) (This x is S and This x is not V)

⇒ (∃x) (Sx • ~ Vx)

The traditional four categorical propositions, A, E, I and O in the predicate calculus are symbolized as:

A: All S in P

⇒ (x) (Sx ⊃ Px)

E: No S is P

⇒ (x) (Sx ⊃ ~ Px)

I: Some S is P

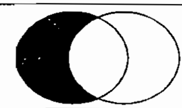
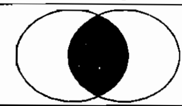
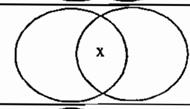
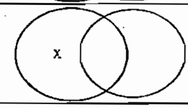
⇒ (∃x) (Sx • Px)

O: Some S is not P

⇒ (∃x) (Sx • ~ Px)

The above symbolization brings out clearly the internal structure of the propositions. Both quantities and qualities of the propositions are made explicit. At the same time more information is given about the characteristics whether they are applied to the same individual or to different people.

The traditional four categorical propositions are symbolized in many ways in the history of logic from time to time. Look at the following four different ways in which these propositions are symbolized.

Proposition	Traditional Accounts	George Boole	Venn Diagram	Predicate Calculus
A	All S is P	$S\bar{P} = 0$		(x) (Sx ⊃ Px)
E	No S is P	SP = 0		(x) (Sx ⊃ ~Px)
I	Some S is P	SP ≠ 0		(∃x) (Sx • Px)
O	Some S is not P	$S\bar{P} \neq 0$		(∃x) (Sx • ~Px)

Singular Propositions

A singular proposition is that which does not contain truth functional connectives like •, ⊃, v, ≡, etc. and also singular propositions do not have words like "all" or "Some" or their equivalents. Subjects in a singular proposition is either a proper name like "Mohan is student of this college" or is a definite descriptive subject like "The present Prime Minister of India." Such propositions are symbolized in different ways.

Note: Small letters like "a", "b", "c" are used for the individuals. Attribute symbol is used towards the left of individual.

Look at the following examples:

1. Ramesh is an engineer

   ⇒ r is E

   ⇒ Er

2. New Delhi is Capital of India

   ⇒ n is C

   ⇒ Cn

3. Rohit is graduate

   ⇒ r is G

   ⇒ Gr

4. Asha is a player

   ⇒ a is P

   ⇒ Pa

5. This boy is tall

   ⇒ b is T

   ⇒ Tb

6. Radha is intelligent and she is hardworking

   ⇒ r is I and r is H

   ⇒ Ir • Hr

   (This shows same person has two different attributes)

Universal and particular propositions are symbolized by using quantifiers and variables:

7. All students are literates

   ⇒ (Whatever x may be) (If x is a student, then x is literate)

   ⇒ (x) (Sx ⊃ Lx)

8. No crow is purple

   ⇒ (Whatever x may be) (If x is a crow, then x is not purple)

   ⇒ (x) (Cx ⊃ ~ Px)

9. Some voters are women

   ⇒ (There is something x) (This x is a voter and this x is a woman)

   ⇒ (∃x) (Vx • Wx)

10. Some applicants are not Indians

   ⇒ (There is something x) (This x is an applicant and this x is not an Indian)

   ⇒ (∃x) (Ax • ~ Ix)

Note: In order to symbolize propositions using quantifiers and variables, it is important that you first reduce the proposition in a standard form, like you had done earlier in Chapter 3A.

11. Who is not wise after the event? Standard form proposition is "All persons are wise after the event."

   ⇒ (Whatever x may be) (If x is a person then x is wise after the event)

   ⇒ (x) (Px ⊃ Wx)

12. He who hesitates fails. Standard form proposition is "All those who hesitate are those who fail."

   ⇒ (Whatever x may be) (If x hesitates then x fails)

   ⇒ (x) (Hx ⊃ Fx)

13. None of the doctors in the city could save him.

   ⇒ No doctor in the city is a person who could save him.

   ⇒ (Whatever x may be) (If x is a doctor in the city then x is a person who could not save him)

   ⇒ (x) (Dx ⊃ ~ Sx)

14. Only brave are winners.

   ⇒ All those who are winners are brave.

   ⇒ (Whatever x may be) (If x is a winner then x is a brave).

   ⇒ (x) (Wx ⊃ Bx)

15. Not all people love cricket.

   ⇒ Some people are not cricket lovers.

   ⇒ (There is something x) (This x is people and this x is not cricket lover)

   ⇒ (∃x) (Px • ~ Cx)

There are propositions where the subject or predicate has more than one characteristic such as:

1. All students and teachers are members of library. Then one propositions is symbolized as:

   ⇒ (Whatever x may be) (If x is either a student or x is a teacher, then x is member of library)

   ⇒ (x) (If x is S or x is T, then x is M)

   ⇒ (x) [(Sx v Tx) ⊃ Mx]

Here 'and' between student and teacher is deceptive. The meaning of the above proposition is that x is either a student or x is teacher, but x is not both student and teacher.

Take another example:

2. All brave soldiers were given medals. This is symbolized as:

   ⇒ (Whatever x may be) (If x is soldier and x is brave, then x is given medal)

   ⇒ $(x)\ [(Sx \bullet Bx) \supset Mx)]$

   Here soldiers and brave go together and hence joined by '•'

3. "Boys who are athletes are strong":

   ⇒ (Whatever x may be) (If x is a boy and x is a athlete, then x is strong)

   ⇒ $(x)\ [(Bx \bullet Ax) \supset Sx]$

4. Mobile phones that are expensive are lost quickly.

   ⇒ (Whatever x may be) (If x is a mobile phone and x is expensive, then x is lost quickly)

   ⇒ $(x)\ [(Mx \bullet Ex) \supset Lx]$

These are propositions where both subject and predicate have more characteristics.

5. All professional athletes are healthy and strong.

   ⇒ (Whatever x may be) (If x is an athlete and x is professional, then x is healthy and x is strong)

   ⇒ $(x)\ [(Ax \bullet Px) \supset (Hx \bullet Sx)]$

6. Children who eat protein are strong and active.

   ⇒ (Whatever x may be) (If x is a child and x eats protein then x is strong and x is active)

   ⇒ $(x)\ [(Cx \bullet Px) \supset (Sx \bullet Mx)]$

7. It is false that some books are expensive.

   ⇒ ~ (There is something x) (This x is a book and this x is expensive)

   ⇒ $\sim (\exists x)\ (Bx \bullet Ex)$

8. It is false that every book is expensive.

   ⇒ ~ (Whatever x may be) (If x is a book then x is expensive)

   ⇒ $\sim (x)\ (Bx \supset Ex)$

Exercise 17

A. Symbolize the following propositions using quantifiers and suggested variables.

1. Carrots are nutritious.
   (Cx, x is Carrot; Nx, x is nutritious)
2. A few diamonds are blue.
   (Dx, x is diamond; Bx, x is blue)
3. No unicorns eat oats.
   (Ux, x is unicorn; Ox, x eats oats)
4. Some dragons do not breathe fire.
   (Dx, x is dragon; Bx, x breathes fire)
5. All cars are useful.
   (Cx, x is car; Ux, x is useful)
6. All Kangaroos are introvert.
   (Kx, x is Kangaroo; Ix, x is introvert)
7. Most of the dogs are intelligent.
   (Dx, x is dog; Ix, x is intelligent)
8. Every soldier serves his country.
   (Sx, x is soldier; Cx, x serves his country)
9. The envious are never happy.
   (Ex, x is envious; Hx, x is happy)
10. Rivers generally run into the sea.
    (Rx, x is river; Sx, x runs into the sea)
11. Every man is fallible.
    (Mx, x is man; Fx, x is fallible)
12. No horse is bipeds.
    (Hx, x is horse; Bx, x is bipeds)

13. Whales are mammals.
(Wx, x is whales; Mx, x is mammal)
14. Majority of the students of this institute are not Indians.
(Sx, x is student of this institute; Ix, x is an Indian)
15. Every scientist is a serious thinker.
(Sx, x is a scientist; Tx, x is a serious thinker)
16. No angels are mortal.
(Ax, x is angel; Mx, x is mortal)
17. All socialists are not communists.
(Sx, x is a socialist; Cx, x is a communist)
18. Actors are generally hard workers.
(Ax, x is an actor; Hx, x is hard worker)
19. Every student of this class can do this question.
(Sx, x is a student of the class; Dx, x can do this question)
20. A few professors are not strict.
(Px, x is a professor; Sx, x is strict)
21. Crows are black.
(Cx, x is crow; Bx, x is black)
22. Only ungrateful people blame.
(Ux, x is ungrateful people; Bx, x blames)
23. No men is reptile.
(Mx, x is a man; Rx, x is a reptile)
24. All horses are animals.
(Hx, x is a horse; Ax, x is an animal)
25. Not every man can swim.
(Mx, x is a man; Sx, x can swim)
26. Only philosophers are scholars.
(Px, x is a philosopher; Sx, x is a scholar)
27. No leopards are gentle.
(Lx, x is a leopard; Gx, x is gentle)
28. Each bird is bipeds.
(Bx, x is a bird; Px, x is biped)

29. Only ticket holders are permitted.
(Tx, x is ticket holder; Px, x is permitted)
30. Every giraffe is herbivorous.
(Gx, x is a giraffe; Hx, x is herbivorous)

B. Symbolize the following propositions using quantifiers and suggested notation:

1. A triangle is bounded by three straight lines.
(Tx, x is triangle; Bx, x is bounded by three straight lines)
2. Vices never bring happiness.
(Hx, x is vice; Bx, x brings happiness)
3. Only those who are unselfish are happy.
(Ux, x is unselfish; Hx, x is happy)
4. Not all schemes of social reform are feasible.
(Sx, x is scheme of social reform; Fx, x is feasible)
5. A few voters are literate.
(Vx, x is a voter; Lx, x is literate)
6. Every rectangle is a parallelogram.
(Rx, x is rectangle; Px, x is parallelogram)
7. Only intelligent persons are scholars.
(Ix, x is an intelligent person; Sx, x is a scholar)
8. All racists are insecure.
(Rx, x is racist; Bx, x is insecure)
9. Every triangle is a polygon.
(Tx, x is a triangle; Px, x is a polygon)
10. Fat people are happy.
(Fx, x is a fat person; Hx, x is happy)
11. Few men are cultured.
(Mx, x is a man; Cx, x is cultured)
12. Mice are rodents.
(Mx, x is mice; Rx, x is rodent)
13. All snakes are not poisonous.
(Sx, x is snake; Bx, x is poisonous)

14. No poachers are law-abiding
(Px, x is a poacher; Lx, x is law-abiding)
15. At least some officers are not corrupt.
(Ox, x is an officer; Cx, x is corrupt)
16. All acids are compounds.
(Ax, x is acid; Cx, x is compound)
17. Every unwanted plant is weed.
(Ux, x is unwanted plant; Wx, x is weed)
18. All those who are happy on earth are virtuous.
(Wx, x is one who is happy on earth; Vx, x virtuous)
19. Few people are scholars.
(Px, x is people; Sx, x is scholar)
20. Birds are not quadrupeds.
(Bx, x is bird; Qx, x is quadrupeds)
21. All sailors are adventurous people.
(Sx, x is sailor; Ax, x is adventurous people)
22. None but healthy persons are recruited in army.
(Hx, x is healthy person; Rx, x is recruited in army)
23. A tiger is not a domestic animal.
(Tx, x is a tiger; Dx, x is a domestic animal)
24. Almost all antibiotics are remedies.
(Ax, x is antibiotics; Rx, x is remedy)
25. None but communists are radicals.
(Cx, x is communist; Rx, x is radical)
26. All living things are perishable.
(Lx, x is living thing; Px, x is perishable)
27. Whales are mammals.
(Wx, x is a whale; Mx, x is a mammal)
28. All peace-loving persons hate war.
(Px, x is peace-loving person; Hx, x hates war)
29. All models are attractive.
(Mx, x is a model; Ax, x is attractive)

30. Many problems are easy to solve.
(Px, x is problem; Ex, x is easy to solve)
31. Some well-informed people are wise persons.
(Wx, x is well-informed people; Px, x is a wise person)
32. Democracy respects human rights.
(Dx, x is democracy; Rx, x respects human rights)
33. No computers are animated objects.
(Cx, x is computer; Ax, x is animate object)
34. Every college student is adult.
(Cx, x is college student; Ax, x is adult)
35. Nations are political entities.
(Nx, x is nation; Px, x is political entity)
36. Not every octopus has a bad temper.
(Ox, x is octopus; Bx, x has bad temper)
37. Some Americans are not materialists.
(Ax, x is American; Mx, x is materialist)
38. Only elephants have trunks.
(E, x is elephant; Tx, x has trunk)
39. A few men are optimistic.
(Mx, x is a man; Ox, x is optimistic)
40. Ram is graduate.
(Gx, x is graduate, r is for Ram)

C. Symbolize the following propositions using quantifiers and variables:

1. Taxpayers are not paupers.
(Tx, x is taxpayer; Px, x is pauper)
2. Rakshit is tall and he is a good sportsman.
(Tx, x is tall; Gx, x is good sportsman, r is for Rakshit)
3. All criminals are maladjusted people.
(Cx, x is criminal; Mx, x is maladjusted people)
4. Basketball players are never short.
(Bx, x is a basketball player; Sx, x is short)

5. Nothing which perishes is divine.
(Px, x perishes; Dx, x is divine)
6. All employees are eligible for social security benefits.
(Ex, x is employee; Sx, x is eligible for social security benefits)
7. A few physicians are humanitarians.
(Px, x is physician; Hx, x is humanitarian)
8. White diamonds are expensive.
(Dx, x is diamond; Wx, x is white; Ex, x is expensive)
9. All football players are active and disciplined.
(Fx, x is football player; Ax, x is active; Dx, x is disciplined)
10. The Cheetahs are the fastest runner.
(Cx, x is cheetah; Fx, x is fastest runner)
11. Girls and women are both delicate and pretty.
(Gx, x is girl; Wx, x is woman; Dx, x is delicate; Px, x is pretty)
12. Some cold-blooded animals are not mammals.
(Ax, x is animal; Cx, x is cold-blooded; Mx, x is mammal)
13. Only ambitious people are successful.
(Ax, x is ambitious; Px, x is people; Sx, x is successful)
14. All floods in the valley are either due to heavy rain or melting snow.
(Fx, x is flood in the valley; Hx, x has heavy rain; Mx, x is due to melting snow)
15. Every eligible person is graduate.
(Px, x is person; Ex, x is eligible; Gx, x is graduate)
16. Many well-tried old methods are best remedies.
(Mx, x is method; Wx, x is well-tried; Ox, x is old; Bx, x is best remedies)
17. A few men and women are learned and wise.
(Mx, x is a man; Wx, x is woman; Lx, x is learned; Sx, x is wise)

18. Most of the employed students are optimistic and forward looking people.
(Sx, x is student; Ex, x is employed; Ox, x is optimistic; Fx, x is forward looking; Px, x is people)
19. Every good law is just.
(Lx, x is law; Gx, x is good; Jx, x is just)
20. Few happy men are rich.
(Hx, x is happy; Mx, x is man; Rx, x is rich)
21. All happy men are healthy and contented.
(Hx, x is happy; Mx, x is man; Ex, x is healthy; Cx, x is contented)
22. All citizens who are eligible to vote are above eighteen years old.
(Cx, x is citizen; Ex, x is eligible; Ax, x is above eighteen years old)
23. Every benign tumor is not curable.
(Tx, x is tumor; Bx, x is benign; Cx, x is curable)
24. A well-fed animal is healthy.
(Wx, x is well-fed; Ax, x is animal; Hx, x is healthy)
25. Birds and fishes are vertebrates.
(Bx, x is bird; Fx, x is fish; Vx, x is vertebrates)
26. Some workers are neither skilful nor hardworking.
(Wx, x is worker; Sx, x is skilful; Hx, x is hardworking)
27. A few students are both intelligent and hardworking.
(Sx, x is student; Ix, x is intelligent; Hx, x is hardworking)
28. Many scientific theories have become obsolete.
(Tx, x is theory; Sx, x is scientific; Ox, x obsolete)
29. Several good things of life are free of cost.
(Tx, x is things of life; Gx, x is good; Fx, x is free of cost)
30. Some books are neither expressive nor informative.
(Bx, x is book; Ex, x is expressive; Ix, x is informative)

31. Every faculty member is effective teacher as well as well-informed person.
(Fx, x is faculty member; Ex, x is effective teacher; Wx, x well-informed; Px, x is people)
32. Sonu and Monu are both students and good sportsmen.
(Gx, x is student; Px, x is good sportsman; s is for Sonu, m is for Monu)
33. No scholar who is in the military has academic freedom.
(Sx, x is scholar; Mx, x is in military; Ax, x has academic freedom)
34. Mountains and river are perceptible.
(Mx, x is mountain; Rx, x is river; Px, x is perceptible)
35. Murders are committed and regretted.
(Mx, x is murder; Cx, x is committed act; Rx, x is regretted act)
36. Evening colleges are either useless or troublesome.
(Ex, x is evening; Cx, x is college; Ux, x is useless; Tx, x is troublesome)
37. Genuine foods contain hydrogen and oxygen.
(Fx, x is food; Gx, contain x is genuine; Hx, x is hydrogen; Ox, x is oxygen)
38. Every graduate is either intelligent or hardworking.
(Gx, x is graduate; Ix, x is intelligent; Hx, x is hardworking)
39. A politician who is elected as MLA is public servant.
(Px, x is politician; Ex, x is elected as MLA; Sx, x is public servant)
40. German Shephards are dogs.
(Sx, x is Shephard; Gx, x is German; Dx, x is dog)

Chapter 15 – Section B

Validity

FORMAL validity of an argument in the predicate calculus is examined in the similar way as has been done in the previous chapter of the formal proof of validity. Rules of Inference are applied here also. But in addition to those rules four more rules are introduced to test the validity of arguments falling in the predicate calculus. These four rules felicitate in reducing the predicate calculus argument in the framework of propositional calculus. The rules are as follows:

1. Universal Instantiation (UI)
2. Existential Instantiation (EI)
3. Universal Generalization (UG)
4. Existential Generalization (EG)

What is meant by Instantiation? This is something which was stated by the traditional logicians in the form of *Dictum de omni et nullo*, that is, whatever is predicated (affirmatively or negatively) universally or particularly to any class, it may be predicted (affirmatively or negatively) to its individuals. For instance, if "All men are mortal" is true then "Ram is mortal" is also true. Similarly if "No men are immortal" is true then "Ram is not immortal" is also true.

Existential Instantiation means that if "Some students are voters" is true then at least one student say "Ram is a voter" is also true. Similarly if "Some students are not employed" is true, then there is at least one student say "Ram who is not employed" is also true.

To test the validity of the following argument:

All students are optimists.

All applicants are students.

Therefore, all applicants are optimists.

We first symbolize it in terms of quantifiers and variables:

(x) (Sx ⊃ Ox)

(x) (Ax ⊃ Sx)

∴ (x) (Ax ⊃ Ox)

There are two ways to proceed further. First by applying the Rule of Universal Instantiation, we reduce the complete argument in the following manner:

Sa ⊃ Oa

Aa ⊃ Sa

∴ Aa ⊃ Oa

'x' which is individual variable is replaced by individual constant 'a'. In this manner the entire argument of the predicate calculus is reduced to propositional calculus by shaking off the quantifiers. Now the formal proof of validity can be constructed with the help of Nine Rules of Inference.

1. Sa ⊃ Oa
2. Aa ⊃ Sa / ∴ Aa ⊃ Oa
3. Aa ⊃ Oa 2nd and 1st by H.S.

Since formal proof of validity can be constructed for the above propositional argument, the corresponding predicate calculus argument is valid as well. For if a formula is formally proved valid in the propositional calculus then it is also formally proved valid in the predicate calculus.

There is another way of constructing formal proof of validity of the above argument. The second method is slightly different from the previous one. In the second method, instead of taking the entire argument as a model of instantiation, we reduce one by one quantifier of the premisses. The quantifiers from the premisses are peeled off from the premisses by using the rules of

UI or EI. This is to reduce the argument of predicate calculus in the formate of propositional calculus. After using nine Rules of Inference and getting what we want to prove, we shift from propositional calculus platform to the predicate calculus by imposing or wearing the quantifiers with the help of UG or EG as the case may be. The above argument is proved valid in the following manner:

1.	(x) (Sx ⊃ Ox)	
2.	(x) (Ax ⊃ Sx)	/ ∴ (x) (Ax ⊃ Ox)
3.	Sa ⊃ Oa	1st by UI
4.	Aa ⊃ Sa	2nd by UI
5.	Aa ⊃ Oa	4th and 3rd by H.S.
6.	(x) (Ax ⊃ Ox)	5th by UG

Take another example:

1. (x) (Sx ⊃ Mx)
2. (∃x) (Sx•Px) / ∴ (∃x) (Mx)

According to 1st method the formal proof of validity is constructed in the following manner:

Sa ⊃ Ma

Sa•Pa

∴ Ma

by applying UI and GI.

1.	Sa ⊃ Ma	
2.	Sa•Pa	/ ∴ Ma
3.	Sa	2nd by Simplification
4.	Ma	1st and 3rd by Modus Ponens

The above argument is formally proved valid in propositional calculus. Therefore, its corresponding predicate calculus argument is also proved valid.

By applying the second method, the argument is proved formally valid as follows:

1. (x) (Sx ⊃ Mx)
2. (∃x) (Sx•Px) / ∴ (∃x) (Mx)
3. Sa ⊃ Ma — 1^{st} by UI
4. Sa•Pa — 2^{nd} by EI
5. Sa — 4^{th} by Simplification
6. Ma — 3^{rd} and 5^{th} by Modus Ponens
7. (∃x) (Mx) — 6^{th} by EG

Exercise 18

A. Construct Formal Proof of validity for each of the following arguments:

1. (x) (Px ⊃ Sx)
   (∃x) ~ Sx / ∴ (∃x) (~ Px)
2. (x) (Lx ⊃ Mx)
   (x) (Mx ⊃ Sx) / ∴ (x) (Lx ⊃ Sx)
3. (∃x) (Px•Sx)
   (x) (Px ⊃ Hx) / ∴ (∃x) (Hx)
4. (x) [(Px•Mx) ⊃ Sx]
   (x) (Px•Mx) / ∴ (x) Sx
5. (x) [Px ⊃ (Mx•Hx)]
   (x) ~ (Mx•Hx) / ∴ (x) ~ Px
6. (x) Fx
   (x) Mx / ∴ (x) (Fx•Mx)
7. (x) (Px ⊃ Qx)
   Pa / ∴ Qa
8. (x) (Px v Mx)
   (x) (~ Px) / ∴ (x) (Mx)
9. (∃x) (Px)
   (x) (Px ⊃ Hx) / ∴ (∃x) (Hx v Lx)

10. (x) [(Fx ⊃ Gx) • (Hx ⊃ Lx)]
    (x) (Fx v Hx) / ∴ (x) (Gx v Lx)
11. (x) (Px ⊃ Sx) / ∴ (x) [Px ⊃ (Px•Sx)]
12. (x) [Fx ⊃ (Gx•Hx)]
    Fa / ∴ Ga
13. (x) (Sx ⊃ Tx)
    (x) (Tx ⊃ Rx)
    (x) (Rx ⊃ Gx) / ∴ (x) (Sx ⊃ Gx)
14. (x) [Rx ⊃ (Tx v Sx)]
    (x) [Sx ⊃ (Ux ⊃ Wx)]
    (∃x) (~ Tx•Rx)
    (x) Rx
    ∴ (∃x) (Ux ⊃ Wx)
15. (x) (Mx ⊃ Hx)
    (x) (Gx ⊃ Mx)
    (x) (Lx ⊃ Gx) / ∴ (x) (Lx ⊃ Hx)

B. Construct Formal Proof of validity for each of the following arguments using suggested notations:

1. All physicians are college graduates.
   All surgeons are physicians.
   Therefore, all surgeons are college graduates.
   (Px, x is physician; Cx, x is college graduate; Sx, x is surgeon).
2. All good lawyers read fine print in contracts.
   Rohit is a good lawyer.
   Therefore, Rohit reads fine print in contracts.
   (Lx, x is good lawyer; Rx, x reads fine prints in contracts; r is for Rohit).
3. All Indians are peace loving people.
   Gujrati's are Indians.
   Therefore, Gujratis are peace loving people.
   (Ix, x is Indian; Px, x is peace loving people; Gx, x is Gujrati)

4. All prisoners are captives.
   No captives are happy.
   Therefore, No prisoners are happy.
   (Px, x is prisoner; Cx, x is captive; Hx x is happy person).
5. All iron objects are metallic.
   All metals conduct electricity.
   So all iron objects conduct electricity.
   (Ix, x is iron object; Mx, x is metallic or metal; Cx, x conducts electricity).
6. All mammals have heart.
   All horses are mammals.
   Therefore, all horses have heart.
   (Mx, x is mammal; Hx, x has heart; Rx, x is horse).
7. All researchers are dedicated.
   Some researchers are insurance persons.
   Some insurance persons are secure.
   Therefore, some insurance persons are dedicated persons.
   (Rx, x is researcher; Dx, x is dedicated; Ix, x is insurance person; S, x is secure).
8. All models are attractive.
   Some models are active in civil right movement.
   All attractive persons are envied persons.
   Therefore, some attractive persons are envied persons.
   (Mx, x is model; Ax, x is attractive; Cx, x is active in civil rights movements; Ex, x is an envied persons).
9. All sailors are travellers.
   All travellers are knowledgeable.
   All knowledge persons are broadminded.
   All broadminded people are confident. Therefore, all sailors are confident.
   (Sx, x is sailor; Tx, x is traveller; Kx, x is knowledgeable; Bx, x is broadminded; Cx, x is confident).
10. An animal is substance.
    All quadruped is an animal.

    A horse is a quadruped.
    All bucephalus are horses.
    Thus all bucephalus are substance.
    (Ax, x is animal; Sx, x is substance; Qx, x is quadruped; Bx, x is bucephalus; Hx, x is horse).

Chapter 15 – Section C

Invalidity

THE invalidity of arguments in the predicate calculus is proved in the similar way as has been done in the propositional calculus proving invalidity by assigning truth values. By using rules of UI and EI the argument is proved invalid for the individual. For instance the invalidity of the following argument is proved in the following manner:

$(x) \quad (Fx \supset Gx)$

$(x) \quad (Mx \supset Gx) \qquad / \therefore (x)\ (Fx \supset Mx)$

By applying UI, the argument is reduced to propositional calculus formate.

$Fa \supset Ga$

$Ma \supset Ga \qquad / \therefore Fa \supset Ma$

Keeping in sequence it is written as follows:

8	6	10	2	9	7	11	1	4	3	5
[(Fa	⊃	Ga)	•	(Ma	⊃	Ga)]	⊃	(Fa	⊃	Ma)
T	T	T	T	F	T	T	F	T	F	F

No error, no inconsistency found, hence the above argument is proved invalid in the propositional calculus. Therefore it is proved invalid in the predicate calculus as well.

2. Take another example:

$(x) \quad (Fx \supset Gx)$

$(x) \quad (\sim Fx) \qquad / \therefore (x)\ (\sim Gx)$

After applying UI, we get the following sequence:

8	5	9	2	6	7	1	3	4
[(Fa	⊃	Ga)	•	(~	Fa)]	⊃	(~	Ga)
F	T	T	T	T	F	F	F	T

No error hence proved invalid.

3. Take yet another example:

$(x) \quad (Px \supset \sim Mx)$

$(\exists x) \quad (Px) \qquad / \therefore (x)\ (Mx)$

replacing the above argument by using individual constant we get the following sequence:

6	4	8	7	2	5	1	3
[(Pa	⊃	~	Ma)	•	Pa]	⊃	Ma
T	T	T	F	T	T	F	F

No error hence proved invalid.

4. See another example:

$(x) \quad (Px \supset Gx)$

$(x) \quad (Gx \supset Mx)$

$(x) \quad (Mx \supset Lx) \qquad / \therefore (x)\ (Lx \supset Px)$

After applying UI, we get the following sequence:

11	7	12	2	13	8	14	3	15	9	10	1	5	4	6
[(Pa	⊃	Ga)	•	(Ga	⊃	Ma)	•	(Ma	⊃	La)]	⊃	(La	⊃	Pa)
F	T	T	T	T	T	T	T	T	T	T	F	T	F	F

The above argument is proved invalid by assigning truth values. Hence its corresponding predicate calculus argument will also be invalid.

5. $(x) \quad (Fx \supset Gx)$

$(\exists x) \quad (Gx \bullet Lx) \qquad / \therefore (\exists x)\ (Lx \bullet Fx)$

After reducing this argument into propositional calculus sequence by replacing individual variable x by individual constant 'a' with the help of UI and EI, we get the following sequence:

11	6	10	2	8	7	9	1	4	3	5
[(Fa	⊃	Ga)	•	(Ga	•	La)]	⊃	(La	•	Fa)
F	T	T	T	T	T	T	F	T	F	F

The above argument is proved invalid by assigning truth values in propositional calculus. Hence its corresponding predicate calculus argument is also proved invalid.

Exercise 19

A. Prove the invalidity of the following argument forms:

1. (x) (Fx v Gx)
(x) (Gx ⊃ Hx)
∴ (∃x) (Hx ⊃ Fx)

2. (x) (Fx ⊃ Gx)
(∃x) (~ Gx)
∴ (∃x) (Fx)

3. (x) (Gx ⊃ ~ Hx)
(∃x) (Fx • ~ Gx)
∴ (∃x) (Hx • ~ Fx)

4. (∃x) (Lx v Mx)
(x) (Mx ⊃ Px)
∴ (∃x) (Px)

5. (x) (Px ⊃ Qx)
(x) (Rx ⊃ Qx)
∴ (x) (Px ⊃ Rx)

6. (x) (Fx ⊃ Gx)
(x) (Gx ⊃ Hx)
(x) (Px ⊃ Hx)
(∃x) (Fx • Tx)
∴ (∃x) (Px • Tx)

7. (x) (Px ⊃ Qx)
(x) (Qx ⊃ Rx)
(x) (Rx ⊃ ~Tx)
(x) ~ Px / ∴ (∃x) ~ Tx

8. (x) (Mx ⊃ Nx)

(x) (Nx ⊃ Lx)
(x) (Hx ⊃ Mx)
(∃x) (Lx)
(∃x) (Mx) / ∴ (x) (Nx ⊃ Hx)

9. (x) [(Px ⊃ Qx) • (Rx ⊃ Sx)]
(x) (Px ⊃ Rx)
(∃x) (~ Px • Rx)
(x) (Mx ⊃ Sx)
(∃x) (Qx) / ∴ (x) (Mx ⊃ Px)

10. (∃x) (Hx)
(x) (Hx ⊃ ~ Lx)
(∃x) (~ Lx)
(x) [(Lx ⊃ (Hx • Px)]
(x) (Px ⊃ Lx) / ∴ (x) (Hx ⊃ Lx)

B. Prove the invalidity of the following arguments using suggested notations:

1. All men are mortal.
All professors are mortal.
∴ All professors are men.
(Mx, x is man; Tx, x is mortal; Px, x is professor).

2. All students are intelligent persons.
Some intelligent persons are voters.
∴ Some voters are students.
(Sx, x is student; Ix, x is intelligent person; Vx, x voter)

3. All capitalists are critical of communists.
Some critical of communists are intellectuals.
Therefore, some intellectuals are capitalists.
(Cx, x is capitalist; Rx, x is critical of communists; Ix, x is intellectual).

4. All socialists believe in nationalization.
All communists believe in nationalization.
Therefore, all socialists are communists.
(Sx, x is socialist; Nx, x believes in nationalization; Cx, x is communist).

5. All scientists are researchers.
Some scientists are not well paid.
Therefore, some well paid are not researchers.
(Sx, x is scientist; Rx, x is researcher; Wx, x is well paid).

6. Some acts of civil disobedience are not violent.
All violent acts are morally unsound.
Therefore, some morally unsound acts are acts of civil disobedience.
(Cx, x is act of civil disobedience; Vx, x is violent act; Mx, x is morally unsound).

7. All materialists are atheists.
Some Hindus are not atheists.
Therefore, all Hindus are materialists.
(Mx, x is materialist; Ax, x is atheist; Hx, x is Hindu).

8. All doctors are psychoanalysts because all psychiatrists are psychoanalysts and all psychiatrists are doctors.
(Px, x is psychiatrist; Dx, x is doctor; Ax, x is psychoanalyst).

9. No military men are expert in social legislation for all for those who trained for war are military men and some of those who are trained for war are expert in social legislation.
(Mx, x is military men; Ex, x is expert in social legislation; Tx, x is trained for war).

10. Since all poets live in the world of dreams and they all are ignorant of the evils of reality, so all those who live in the world of dreams are ignorant of the evils of reality.
(Px, x is poet; Ix, x is ignorant of the evils; Dx, x lives in the world of dreams.)

Reference

Basson, A.H. and D.J. O'Connor, *Introduction to Symbolic Logic*.

Part III

Chapter 16

Induction

INDUCTION like deduction is a form of reasoning; it is a type of inference in which from some observed instances a conclusion is drawn about the unobserved instances of the same kind. Though the premisses do not necessarily imply the conclusion in induction, yet the premisses are good reason for drawing the conclusion. No empirical science, natural or social, which aims to describe nature, world, or society, can do without induction. In fact all sciences, except pure mathematics and formal logic, are making extensive use of inductive inferences.

In induction, on the basis of some observed instances, a conclusion about the unobserved instances is drawn. For instance, on the basis of observed crows found to be black, one draws a conclusion that "All crows (observed as well as unobserved) are black". From observing Ram is mortal, Mohan is mortal, Sita is mortal and so on, one concludes "All men are mortal". The formal structure of this type of inductive reasoning can be symbolically expressed as:

x_1, x_2, x_3 . . ., have the characteristics of A. Therefore probably all xs have the characteristics of A.

The above cited examples of inductive reasoning can be elaborated logically as follows:

(1) Crow x was observed and it was black.

"	y	"	"	"
"	z	"	"	"
!	!	!	!	!

Therefore, probably all crows are black.

(2) Ram is mortal.
Mohan is mortal.
Sita is mortal.
! !
!

Therefore, probably all men are mortal.

From this, one may get the impression that induction is a logical process in which a universal conclusion is supported on the basis of some observed and limited number of instances. But this is not always true. Traditionally, it was understood that a deductive inference (reasoning) goes from general premisses to the particular conclusion, and an inductive inference (reasoning) goes from the particular premisses to the general conclusion. Look at the following illustrations:

Deductive reasoning:
All men are mortal.
Ram is a man.

Therefore, Ram is mortal.

Inductive reasoning:

Metal x expanded on heating.
Metal y expanded on heating.
Metal z expanded on heating.

Therefore, probably all metals expand on heating.

Though this distinction between deduction and induction (that is, in deduction particular conclusion is drawn from universal premisses and in induction universal conclusion is drawn from particular premisses) holds good in some cases (for instance in forming empirical inductive generalizations like "All men are mortal", "All crows are black", where we move from particular evidences to universal truths), but there are instances both in deduction as well as in induction where this 'traditional dichotomy' does not 'fit'. In deductive inferences, there are instances where universal conclusions are inferred from the universal premisses. For example:

All men are mortal.
All employees are men.

Therefore, all employees are mortal.

In induction also sometimes we move from particular to particular. For instance:

x is a citizen and a voter in Indian general election.
x is a citizen and a voter in Indian general election.
z is a citizen

Therefore, probably z is a voter in Indian general election.

The most acceptable difference between deduction and induction, nowadays, is that deductive is "necessary" or demonstrative inference, whereas inductive inference is merely "probable". The conclusion "is claimed to follow" necessarily from the premisses in deductive inference, whereas in inductive inference, the conclusion is not "claimed to follow" necessarily (but only with some degree of probability) from the evidences.

The formal sciences like pure mathematics and formal logic are often described as deductive in nature. The reasoning that goes on in pure mathematics and in formal logic is purely deductive. The empirical sciences like physics, chemistry, biology, etc. (all natural sciences), and also political science, economics, history, psychology, etc. (all social sciences) are often described as inductive, for the reasoning involved in them is inductive in nature. But this is wrong. The empirical sciences whether natural or social are not purely inductive. The deductive reasoning also occurs in the empirical sciences as well. The scientists move from empirical data to theories and laws. It is, thus, wrong to think that empirical sciences are totally inductive in nature. Even theoretical sciences (for instance pure physics) cannot do away with sense data. There is an empirical cushion at their back. They need empirical dimension or matrix to verify the theories. Arthur Pap says, "there is no other way of testing an empirical hypothesis — especially one of highly theoretical character, such as the hypothesis of universal gravitation, or the atomic

hypothesis, or the gene theory of biology — than by deducing from it directly testable consequences."[1] This shows that "purely inductive sciences" are not possible, though there are "purely deductive sciences" (pure mathematics and formal logic). All inductive sciences (empirical sciences based on experiences and sense data) make use of inductive as well as deductive processes.

Before we go further, let us see what exactly is meant by induction and what is an inductive inference? According to J.S. Mill, "Induction is the name given to the operation of the mind, by which we infer that what we know to be true in particular case or cases, will be true in any other case or cases of a similar kind. . . ."[2] In other words induction is a process by which we conclude that what is true of certain individuals of a class is true of the whole class, or that what is true at certain times will be true in similar circumstances at all times.

Induction is "concatenated" process in the sense that the results of induction that have already proved successful become the criteria of soundness of inductions yet to come. There is then an 'inductive leap' in this logical process because we move from one level to another level, from observed to unobserved instances. One jumps from one level of quantity to another level of quantity, from one platform to another platform. That is why the results of inductive inferences are only probable though the degree of probability may be very high in certain cases. The relation between evidences and conclusion is not of necessity but rather of suggestion.

An inductive logician has twofold tasks:

(i) to collect evidences (premisses) or sense data from experiences and observations, and

(ii) to draw a conclusion from these evidences (premisses).

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1. Arthur, Pap, *An Introduction to the Philosophy of Science*, p. 139.
2. J.S. Mill, *A System of Logic*. Appendix A, "Of the various grounds of Induction", p. 1103.

In deductive logic, on the other hand, a logician undertakes just one task, that of drawing the conclusion from the given premisses in accordance with the logical rules. A deductive logician does not hunt for the premisses. They are given to him. He is not even bothered about the actual logical status (truth value) of the premisses either. A deductive logician takes for granted the truth value of the premisses. He is interested merely in the fact that from the given premisses what necessarily follows. He is thus interested only in the 'formal consistency'.

An inductive logician, however, is not so fortunate. He gathers the evidences through experiences and observations. The observed instances serve as evidences (premisses) and from that he draws the conclusion. Moreover, in induction all the premisses are materially true because they are experienced instances whereas in deduction valid reasoning may even have false premisses. The form of an argument is the sole factor in deciding the validity of an argument in deductive reasoning. The material truth of the propositions has little role to play. In inductive reasoning, however, the evidences as well as the conclusion are materially true. Both form and matter of an inference play equal role in the evaluation of an inductive argument.

No inductive argument, however, is 'valid' or 'invalid'. The evaluative notions of validity and invalidity are applied to deductive arguments only. A deductive inference is assessed as either valid or invalid. An inductive inference is neither of them. An inductive argument is evaluated by degrees such as weaker or stronger, appealing or non-appealing, convincing or non-convincing, etc. depending on the strength of the evidences.

The soundness of an inductive argument depends on the strength of the evidences in relation to the conclusion. For example "90 per cent of alcoholics have an unhappy family life, and that x is an alcoholic, therefore, it is highly probable to infer that x has an unhappy family too" has a very high 'truth frequency'. The argument thus is strong and sound. On the other hand, in the inference "45 per cent of post-graduate students miss their

classes and x is a postgraduate student, therefore, x misses his classes too", the 'truth frequency' is very low. The inference thus is weak, less appealing and less convincing. It is quite possible that x is a postgraduate student and still not missing classes. **There is no contradiction in accepting true premisses and false conclusion in inductive inferences. But in the deductive inferences it is impossible to have all true premisses and a false conclusion.**

The inductive procedures (which the twentieth-century logicians have accepted) originated from Francis Bacon[3] in sixteenth century, and because of this Bacon is called "Founder of Modern Inductive Philosophy". He is of the opinion that deductive procedures of Aristotelian syllogism cannot be applied to the principles of sciences. It is in the inductive procedures the scientific methodology is to be justified. J.S. Mill,[4] in the nineteenth century, further contributed to the modern inductive logic, initiated by Bacon three hundred years back. The experimental methods of causation (about which you will learn in later chapters) are formulated by J.S. Mill.

The relationship of induction and deduction always remained a favourite topic for the logicians of all ages. Aristotle and Scholastic logicians saw induction as antithetical to syllogism and to deduction. The formal logicians like Stanlay Jevons,[5] in the

3. Francis Bacon (1561–1626) was English statesman and philosopher of science. Though strictly speaking he himself was not a scientist, he had no laboratory and he made no new discoveries, yet he was "the prophet of modern science" for he gave method and inspiration to it.
4. John Stuart Mill (1806–1873) was the most influential philosopher in the English-speaking world during the nineteenth century. He was an economist, and an administrator too. He is generally held to be one of the most effective spokesmen for the liberal view of man and society.
5. William Stanley Jevons (1835–1882) was a British logician and an economist. He has written many books on logic. His most important contribution to scientific methodology is an account of his "celebrated logical machine".

nineteenth century, tried to prove the supremacy of deduction over inductive procedures while the informal or material logicians such as Francis Bacon and J.S. Mill thought the inductive procedure is more fundamental than deductive ones. Jevons calls induction and deduction as the "inverse process" (induction is the converse of deduction). Francis Bacon calls "induction as ascending process" and "deduction as a descending process".

The inductive logicians tried to prove the supremacy of induction while the deductive logicians were advocating just the opposite. But induction and deduction should not be viewed as to which is more fundamental between them, rather they should be seen as various parts of the same game. They are not antithetical, contradictory or opposed to each other but supplementary and complimentary to each other. Just as in a relay race, the starting point of one player is a terminus for another, similarly induction and deduction differ in their "starting point". The terminus of induction (generalizations) serve as the starting point of deduction. Race seems to be started by induction, and deduction takes over later on. The conclusion of induction serves as the premisses in the deductive reasoning. Both induction and deduction, however, investigate and provide rules to evaluate whether the premisses are adequate evidences to support the conclusion or not.

Types of Induction

Inductive inferences occur in a large variety of forms. Out of them four types of inductive inferences are recognized in the elementary logic. They are:

(1) Induction by simple enumeration.

(2) Induction by complete enumeration.

(3) Induction by analogy

(4) Scientific induction.

- Induction
  - Scientific
  - Unscientific
    - Enumerative (counting)
      - Imperfect induction (induction by simple enumeration)
      - Perfect induction (induction by complete enumeration)
    - Analogical

1. *Induction by simple enumeration*: The simplest and most popular form of induction is induction by simple enumeration. Here generalizations are inferred or made simply on the basis of observed instances. The observed instances are found to possess a particular quality and on that basis we infer that the unobserved instances also possess the same quality. For instance, the crows are observed to be of black colour and men are observed to possess the quality of mortality from that the conclusions "All crows are black" and "All men are mortal" are drawn, respectively. However, no connection or relation is established between being a crow and of black colour, man and mortality. Our past experiences and observations serve as the evidences, they serve as the good reasons to accept the generalizations.

The larger number of conformable or favourable instances, and absence of any contradictory, unconformable, or unfavourable instance is the criteria of dependable induction in this kind of reasoning. "Random Sampling" of the phenomena under investigation increases the reliability of the conclusion. There is a tendency, a 'natural impulse' in a man to expect that if certain phenomenon is occurring repeatedly, then the same phenomenon will occur in the same circumstances in future also. The more are the number of instances, the greater is the probability of induction provided no negative or contrary instance is found. "All crows are black" is true as long as no crow of any other colour is found.

There are, however, many drawbacks in the induction by simple enumeration. Since the induction of this type is based on

simple counting of instances alone, there is a possibility of committing a fallacy of 'hasty generalization'. A single negative instance is enough to destroy the hard earned generalization. One non-black coloured crow is enough to make the generalization "All crows are black" a false generalization. Thus a generalization cannot be proved with certainty by innumerable finite instances. That is why Francis Bacon thought, induction by simple enumeration is puerile [childish], for it is always exposed to the danger of contrary instances.

In spite of all that, the number of observed instances are somehow treated as a mark of reliability in common induction, the more the instances the better or more reliable is the conclusion. However, in certain cases a very large number of instances are not enough to establish a reliable generalization whereas in some other cases a very few instances are enough to establish a reliable generalization. For example, to learn that fire burns and hurts, one dares not to have put his hand in fire again and again. No one knows why and how in some other cases, a few instances are enough to establish a generalization. An experimental physicist who has established through one or two samples of a certain metal melting at such and such a temperature under standard pressure confidently generalizes that all samples of that metal have the same melting point. But a naturalist will not generalize with the same confidence that all members of a given species of birds have the same colour as the one or two observed birds.

2. *Induction by complete enumeration*: An induction by complete enumeration or "perfect induction" as is termed by Aristotle, is a type of reasoning in which every instance or every event has been observed and examined, and then the conclusion is drawn. For example, the conclusion "All the students of this class are Indian citizens" is drawn after examining each and every student of the class. Also the conclusions like "All books of this library are stamped and labelled", "All apples of this container are rotten", "All ladies of this family are educated" are instances of induction by complete enumeration.

Induction by complete enumeration, however, is possible in those cases where the total number of instances can be counted or observed. It is clear that induction of this type is applicable only in few cases. Most of the inductive reasoning done by a common man and by a scientist is not of this type. It is thus of little use to a logician also. He is interested in "incomplete induction" where unobserved conclusion is drawn from a few observed instances. There is an "inductive leap", a jump from observed instances to unobserved conclusion in an incomplete induction which is missing in "perfect induction". The conclusion of a complete induction is like a total "summary of the facts".

3. *Induction by Analogy*: Most of our daily arguments are in the form of analogy. Analogy is a form of inference in which similarity of characteristics possessed by the evidences (premisses) is the basis of the conclusion. Analogy can best be understood with the help of examples. For instance, I had read three books say A, B and C written by an author say X. I enjoyed reading all of them. Next time when I want to read a book, I will most probably pick up a book say D by the same author hoping I will enjoy reading the next book also. But it may be possible that the new book D turns out to be totally boring and uninteresting. Thus the results of analogical arguments are not certain. They are merely probable. Like induction by simple enumeration, an analogical argument is neither totally correct nor totally incorrect. They are rather stronger or weaker, appealing or non-appealing.

Look at another illustration of the use of analogical argument. Four girl students say X, Y, Z and K resemble in respect of having good academic record (R), good personality (P) and good conversational ability (C). X, Y, Z all cleared MBBS entrance test. From the above two premisses it follows that K will also clear the MBBS entrance test. The student K resembles X, Y, Z students in all relevant aspects and these resemblances are the basis to predict that the girl K will qualify for the MBBS test. But it may be possible the girl K may not qualify the test at all in spite of having a good academic record and good personality.

Analogical arguments are widely used by the common people in daily life. A scientist also makes use of analogical arguments very often in discovering casual relationships among events and state of affairs. There are many scientific laws and theories which were discovered and suggested by the analogy. For instance, the function of human body served as the modal for making several machines. The analogy established between the anatomy of frog and anatomy of human body helped doctors to find the causes of various diseases. The planet Mars resembles the earth in certain respects say X, Y, Z, etc. The earth has an additional characteristic of inhabitation also, therefore, probably Mars too has the characteristic of inhabitation.

There are different uses of analogy. A poet makes use of analogy to describe beautiful landscape in order to make his description more lively. A scientist also makes use of analogy to explain some unintelligible phenomenon. Scientist's use of analogy is explanatory. A logician, however, is interested in the analogy neither as description nor even as explanation. He is interested in the analogy only as an argument.[6]

Symbolically an analogical argument can be expressed as:

A, B, C, D are entities having the characteristics X_1 X_2 X_3 X_4.

A, B, C also have another characteristic say X_5.

Therefore, probably D also has the characteristic X_5.

Let us express the example of reading books given above in the argumental form. A, B, C, D are entities or objects among whom the analogy is established. In the present case analogy is established among four entities, that is, four books.

Four books (A, B, C D) have the characteristics of X_1, X_2, X_3, X_4.

A, B, C have another characteristics X_5.

Therefore, D also has the characteristics of X_5.

6. Cf. Irving M. Copi, and Carl Cohen, *An Introduction to Logic*, (Ninth Edition), p. 454.

X_1 - being a book
X_2 - written by a particular author
X_3 - being a detective novel
X_4 - published by a particular publisher
X_5 - interesting book that gave me a feeling of good reading.

Like induction by simple enumeration, an analogy is more reliable and stronger if the number of similarities among things or objects are more. But similarities between the instances should be important and relevant. For instances, if I establish analogy among books (which I liked reading) on the basis of their cover and binding, then this is a irrelevant similarity. For, I enjoyed reading books A, B, C, D because of their style and content, and not because of their appearances and bindings. The force of an argument from Analogy depends "on the character of identity and not on the amount of similarity." We must weight "the points of resemblance rather than count them." Thus in analogical argument, like induction by simple enumeration, "number is not safe guide." The similarity among the entities should be relevant and one relevant similarity is far superior than many irrelevant similarities. But the important question is, how do we know that a similarity is relevant or irrelevant? The characteristics by themselves do not carry tags suggesting relevant or irrelevant characteristics. A fallacy in analogical reasoning often occurs due to the confusion between essential and non-essential characteristics, between relevant and irrelevant characteristics. A good and sound analogical argument is that in which the conclusion is drawn on the basis of sound, careful and comprehensive comparison of the points of resemblances among objects and events.

The relevance of characteristics is to be explained in terms of causation though the argument by analogy itself does not establish a causal relationship between the events or phenomena. And in this respect analogical induction remains at the level of

unscientific induction. Analogy like induction by simple enumeration can suggest a causal relationship between phenomena, and thus is useful for scientific researches. It is, "a guide post pointing out" the direction in which more rigorous investigations should be prosecuted. Analogy is thus an important source of making hypothesis on which scientific researches are based. It is, therefore, a milestone in the direction of scientific induction.

4. *Scientific induction*: This is the best and most reliable form of induction. The aim of scientific induction is to establish natural universal laws on the basis of observations and experiments. It does so by establishing a causal relation between the two phenomena or facts. A scientific induction does all that what an induction by simple enumeration does but it goes further. It establishes a cause and effect relationship between events and facts. If induction by simple enumeration is stamped with causation, it becomes scientific induction. Thus establishing the causal relation between phenomena is the distinguishing mark of scientific induction.

A scientist, and also a person with scientific bend of thinking, always inquires why two facts say A and B always occur together. He answers the queries like why two particular phenomena say A and B always occur together, why with increase in heat, the mercury rises. A scientist looks and hunts for a causal order in the nature, whereas a common man hardly bothers to know why two phenomena A and B occur together. For instance, an experienced and old farmer on the basis of his experiences and observations states that climate A is good for the crop X; but young agricultural engineer says this on the basis of his knowledge of causal relationship. He explains that the climate A is good for crop X because climate A is cause or part of cause of good harvesting of crop X.

Thus, whereas the causal relationship is the essence of scientific induction, induction by simple enumeration is satisfied with observations and samplings alone. A scientist aims to produce

ordered, and organized systems of nature and society. The facts are put in the order of cause and effect. From this, however, one should not gather the idea that a scientist can do without samplings and conformable instances. But whereas in induction by simple enumeration number of observed instances is criteria of its reliability, in the natural sciences the situation is different. Sciences like chemistry or physics do not so heavily depend on the adequate number of instances. The social sciences, on the other hand, require a substantial number of observed instances. Many times you come across the social researchers collecting data by giving questionnaires to people and asking the required information from them. This they do to get the samples, or the data, for drawing conclusions in the form of generalizations.

It is very wrong to think that induction by simple enumeration is of no use to a scientist. It seems that the race in the inductive reasoning is started with induction by simple enumeration and is completed by scientific induction. In other words, both scientific induction and induction by simple enumeration started race together, from the same starting point, but whereas the latter stops in the mid way, the former goes further and establishes the causal relationship between phenomena. Induction by simple enumeration though inferior than the scientific induction, yet it is a 'stepping stone' for the scientific induction. The scientific induction does everything what induction by simple enumeration does, but it does something more. It establishes a causal relationship among the phenomena. Induction based on the causal relationship becomes more reliable and the probability of its conclusion increases subsequently.

Though the induction by simple enumeration and induction based on analogy are not forms of scientific induction, yet they suggest a direction in which more serious and fruitful researches can be carried out by the scientists. In this sense, non-scientific inductions are assets to scientists.

However, the aim of every type of induction is to establish a real, and materially true proposition. Real propositions are fruits

of knowledge and they are opposed to verbal analytical propositions. The propositions like "All bachelors are unmarried", "The sum of all angles of a triangle is equal to 180°", are analytical propositions. The predicates of such propositions do not give any new information. They are only formal generalizations and are of a priori nature. But the propositions like "All solid objects are heavy", "All metals when heated expand" provide some new information, some new knowledge. An inductive reasoning thus provides empirical, factual, materially true synthetical propositions.

The main question which worries the logicians about induction is, how are we justified in stating that since X has worked in past, it will also work in future? Since all the metals when heated got expanded, how are we justified in saying that in future metals when heated will expand. It seems there is no formal ground on which inductive inferences can be justified. They, however, can be justified in the matrix of '**uniformity of nature**' and '**universal law of causation**' which are informal grounds of inductive reasoning. 'Uniformity of nature' and 'Universal law of Causation' are necessary postulates presupposed by every inductive reasoning. Mill, Keynes, David Hume and other eminent empirical philosophers assert that inductive inferences are possible only on the assumption of these general postulates.

The law of 'uniformity of nature' broadly states that if certain phenomenon behaves or acts in a certain manner in the past, then in future the same phenomenon will behave exactly in the same way. If water quenched our thirst or fire burns us, then in future water will quench our thirst and fire will burn us; though it is **not rational** to believe so, but it is certainly **reasonable** to do so.

The scientific induction is based on the axiom of the uniformity of nature that the "same" cause is always and everywhere followed by "same" effect. If a set of certain conditions p, q, r has been followed by an effect X, then the same set of conditions are

followed by X anywhere anytime. Besides the uniform occurrences of cause and effect events, **there is an assumption in all scientific reasonings that every event has a cause**. The scientists have taken for granted that nothing occurs by chance. Every change in the universe is due to some cause.

What puzzles the logicians most is how are these postulates of the "uniformity of nature" and the "universal law of causation" acquired? What is the logical status of these postulates? Are they self-evident and a priori, or are they empirical generalizations? These postulates are neither self evident nor a priori analytical propositions because the truth of analytical propositions is determined by the meaning of the terms used in the propositions. The significance and truth of these postulates are not based on the meaning of the terms used in these postulates either. Only through experience and observation we know nature acts in uniform order. Similarly, it is only by experience and observation we know that every event has a cause. These postulates then are justified through the uniform occurrences of phenomena; but earlier we have seen uniform occurrences of phenomena is justified in terms of postulates. It seems we are reasoning in circle and commit the fallacy of *Petitio principii*.

This may compel us to think that induction is unjustified process of reasoning. But the situation is not that hopeless. Induction can be justified in terms of our mental faculty of accepting that future resembles the present. There is a "natural impulse" in a man to expect that future behaves like the present and past, though at the same time it is wrong to aspire for hundred per cent certainty in this prediction.

Chapter 17

Causation

To establish the causal relationship is the distinguishing mark of the scientific induction. Philosophers, logicians and scientists have defined causation differently according to their requirements. Philosophers deal with the theme of causality which mainly attempts to answer the questions about how or why events happen. The concept of causality is closely related to the problem of determinism and free will. Determinism states that strict causal laws govern all physical and natural events, and even human actions are governed by them.

A logician looks to cause and effect relationship as reasoning in disguise. Cause or parts of a cause are premisses, and effect is conclusion. A logician investigates the legitimacy and sufficiency of the premisses with respect to the conclusion. In this context, a logician investigates with respect to the conclusion. He investigates two questions regarding causation.

(i) What is meant by a cause?

(ii) How a causal relationship is established?

The present chapter deals with the first of these questions and the following chapter with the second.

Causal connection is a relation of invariable succession and hence stronger and stricter connection than merely correlation. Causal relationship implies succession in time. Cause is antecedent, and hence precedes the effect. Effect is consequent, and hence follows the cause. In the time sequence, it is the cause which occurs first and the effect later on. The time interval between cause and effect may be very less but the gap none the less is there. Because of this a cause is different from co-existence.

In the proposition "All crows are black" crow and black colour co-exist. One is not the cause of other.

Though cause is antecedent event, but not every antecedent is the cause or part of the cause of an effect. If x is cause of y, then certainly x precedes y; but to say that since p precedes q, therefore, p is definitely cause of q, is wrong. This kind of thinking leads to the fallacy of *Post hoc; ergo propter hoc* about which you will study in the last chapter of the book. For example, to say that since the hot sun was shining when the house burnt, therefore, the sun is the cause of the house burning, is wrong. For the house can burn in night also when there is no sun. Thus only **invariable and unchangeable antecedent** can be the cause of an effect. But mere invariability of antecedent is also not enough to be a cause of an effect. Day is invariable antecedent of night, and similarly night is invariable antecedent of day, but neither day is cause of night nor night is cause of day. Rather they (day and night) are effect of the rotation of the earth around the sun.

Cause must be invariable as well as **unconditional antecedent** of an effect. Unconditionality of antecedent event means the antecedent is self-sufficient to produce an effect. The antecedent event when capable enough to give rise to effect by itself, then it is called unconditional, for it does not need anything more to give birth to effect. Besides being unconditional and invariable, cause must also be **immediate or proximate antecedent** of an effect. Scientists and logicians are not interested in the remote causes. The conditions which precede cause and which have indirect bearing upon the effect are called "remote, mediate or predisposing causes". For instance in the following case:

A is cause of B.
B is cause of C.
C is cause of D.
D is cause of E.
Therefore, A is cause of E.

A is remote cause of E; B and C are also remote causes of E, though less remote than A. Only D is immediate or proximate cause of E. Logicians and scientists are interested merely in proximate causes. **Cause, thus, is defined as invariable, unconditional, immediate antecedent of an effect.**

It is truistic to say that scientists aim to discover the causal connections. To a scientist the principle of causality is a "guiding principle", and because of this his conception of cause is very precise and exact. A scientist asserts exact propositions of the form "Whenever A occurs in the presence of $C_1 \bullet C_2 \ldots$, A is followed by B".[1]

To a scientist, cause means conditions, factors, components responsible for producing an effect. There is a fundamental assumption in the study of natural and social phenomena that events just do not occur, they occur only under certain conditions. Cause thus is defined in terms of conditions by the scientists. Two types of conditions, necessary and sufficient are recognized by them. P is necessary condition for Q, if Q never occurs in the absence of P. P is sufficient condition of Q, if every occurrence of P is accompanied by the occurrence of Q.

A necessary condition is that whose absence can prevent the effect to occur, but its presence cannot guarantee the effect to occur. Necessary or positive conditions are present when the effect occurs but mere presence of one necessary or few necessary conditions are not enough to produce the effect. To follow I.M. Copi's example of fire,[2] the presence of oxygen, fuel and ignition are three necessary conditions but presence of any one, or two do not ensure the effect to occur. For instance oxygen is present here, fuel is also present but there is no fire here because third necessary condition, that is, ignition is missing.

Sufficient condition is the sum total of all necessary conditions. Conjunction of all necessary conditions imply sufficient condition.

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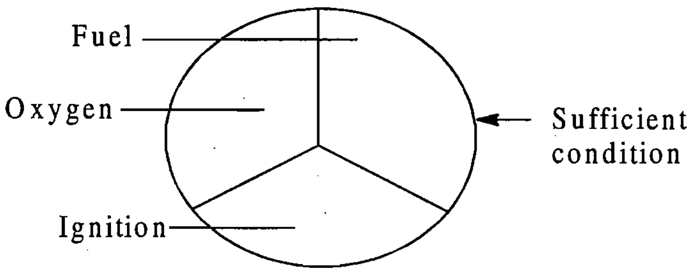
1. Arthur Pap, op. cit., p. 260.
2. Irving M. Copi, and Carl Cohen, op. cit., p. 480.

As soon as the sufficient conditions occur, the effect event also follows without any fail. For instance, fire takes place only when all the three necessary conditions oxygen, fuel and ignition are present at the same time at the same place.

A scientist defines "cause" in terms of necessary conditions when he wants to prevent something unpleasant. In order to extinguish fire at an undesired place, it is not required that all the necessary conditions (for fire to take place) are to be withdrawn. Even if one of the necessary conditions is removed, the effect vanishes. When the wells of oil were burning in Kuwait, (when Iraq attacked Kuwait in early 1990s), the supply of oxygen was stopped in and around the oil wells to extinguish fire. By stopping the supply of oxygen to the wells, fire was extinguished though other two conditions fuel and ignition were there.

A scientist defines cause in terms of sufficient condition when his aim is to produce something desirable. The presence of sufficient condition ensures the presence of effect. Wherever and whenever fire is required and desired, oxygen, fuel and ignition all should be there.

Necessary and sufficient conditions are not isolated. They are related to each other as parts to whole. Whereas sufficient condition should be treated as a whole, a necessary condition is merely a part of the whole.



The circle represents whole and hence stands for sufficient condition. Parts of the circle represent necessary conditions. One

can equate cause with the sufficient condition, but one cannot equate a cause with necessary condition.

The relation between cause and condition is of peculiar type. If by condition we mean sufficient condition then cause can be equated with condition. But if, on the other hand, by condition we mean a necessary condition, then cause cannot be equated with condition. It is very simple; whole is equated with whole and part with part; whole cannot be equated with part.

Plurality Theory of Causation

The scientific interpretation of causation (discussed above), has an important characteristic that the "same" cause produces the "same" effect. If there is slightest change in the cause event, then effect event also changes accordingly. This scientific interpretation of causation that the "same" cause produces "same" effect, however, differs from the views held by certain logicians, like J.S. Mill, who thinks there can be *more than one cause* which produce the *same effect*. There can be several causes, "several logically independent sufficient conditions" which produce the "same" effect. This theory is called **plurality theory of causation.** "Many causes may produce mechanical motion; many causes may produce some kinds of sensation; many causes may produce death."[3]

The causes of unnatural death are several such as accident, illness, bullet injury, poison, burns, etc. The effect remains the same. Take another example, there are many sources of light say electricity, fire, moon, sun, candle, etc. Take yet another example of poor cultivation of crops. The crops can fail due to various reasons. The flood can destroy the crops, the insects may damage it, the drought can spoil it, but the result is single, that is the crops had failed.

For a common man and on the level of common sense understanding the plurality theory of causation seems to be

3. J.S. Mill, *A System of Logic*, bk. 3, ch. 10, sec. 1.

correct and acceptable. But if we analyse the situation a little further and go in depth, then the plurality theory of causation will be found incorrect and unattainable. In all the examples cited above, the causes were analysed and separated but effect events were not analysed properly. The causes are labelled as C_1, C_2, C_3, etc. but effect event was wrongly presumed to be identical. We have wrongly thought effect is one and the same. Whenever there is slightest change in cause event effect, event also changes. Death due to bullet injury is of different kind than the death due to poisoning. If all the deaths would have been of the same kind, then why post-mortem is done to determine the right cause of death.

The crops failed due to flood is of different kind than the crops failed due to insects, or lack of proper care. Similarly sunlight is different from moon light. Candle light is of a unique type, and different from the rest of other types of lights. As the sources of light are different, nature of light is also different. Against the plurality theory of causation, the **unique theory of causation** ("same" cause produces "same" effect) appears to be more logical and consistent.

C_1 --------------> E_1

C_2 --------------> E_2

C_3 --------------> E_3

The unique theory of causation is also compatible with scientific interpretation of causation which is in terms of conditions. A set of certain conditions produces a certain effect, and the slightest variation in those conditions makes difference to the effect event.

The concept of cause as the set of certain conditions responsible for producing an effect, however, no longer fascinates a scientist. This conception of causation in terms of conditions is replaced by the latest concept of "functional dependence". The word "function" is widely used in mathematics. In an effort to be more accurate and certain, the scientists express causal relationships in terms

of mathematical equations. Consequently, scientific laws are formulated in mathematical terms and mathematical framework. You are already familiar with the notion of function as we have made use of it earlier in symbolic logic. When the value of one variable depends on the value of another variable, then the former variable is called function of the latter variable. For example, when the value of Y is determined by the value of X, then Y is called function of X. Since the occurrences of certain events depends on the occurrences of certain other events, the scientists nowadays express the causal relationships in terms of "functional dependence".

Chapter 18

J.S. Mill's Experimental Methods

In the previous chapter you have learnt that to discover a causal relation among facts is the primary aim of a scientist. The crucial question now is how to discover a causal relation? How do we know that A is the cause of B? How does a scientist arrive at the conclusion that the certain conditions say X is the cause of Y phenomenon?

The logicians believe that there are methods which help in revealing the causal connections in the nature. Francis Bacon, in the sixteenth century, was the first such logician who emphasized in the efficiency and sufficiency of these methods.

The methods of discovering causal relationships though were first suggested by him, but it was J.S. Mill who formulated and elaborated them systematically in the nineteenth century. He (Mill) for the first time clearly defined these methods and highlighted their importance in scientific investigations. The formulation and structure of the methods, as we see them today, were given by him and he called them "methods of experimental inquiry". The experimental methods given by him are as follows:

(1) Method of Agreement
(2) Method of Difference
(3) Joint Method of Agreement and Difference
(4) Method of Residues
(5) Method of Concomitant Variation

Before we discuss in detail each of these methods, it is necessary to understand certain presuppositions which Mill made regarding the causal relationship. First, Mill's methods of experimental inquiry take for granted the truth that wherever and whenever cause is present, effect is also present. Second,

whenever and wherever cause is absent, effect is also absent. Third, whenever cause varies (increases or decreases) effect also varies accordingly. Fourth, whatever is the cause of something, the same cause cannot produce something else. Mill's experimental methods are intelligible only in terms of these characteristic presuppositions of causation.

Let us now discuss each of Mill's methods in detail.

Method of Agreement

The method of agreement is the most popular of all the experimental methods. It is used by common men, scientists and philosophers alike. The basis of the method of agreement is the occurrence of a certain phenomenon repeatedly. In our experiences we observe some facts, occur simultaneously. Whenever one fact, say *A* occurs, another fact say *a* also occurs. The togetherness of two facts *A* and *a* leads human beings to relate them casually. The human mind becomes so accustomed and conditioned by observing togetherness of the two facts that next time whenever one sees *A* event, he expects *a* to follow. The strength of the method of agreement depends on the large number of instances. The greater are the instances, the more reliable is the causal relation.

The method can best be understood with the help of examples. J.S. Mill's example for the method is as follows:

In our experiences we have always found that whenever fatty oil or a fat was mixed with an alkaline substance, a soap substance was produced. This has been the case in all sorts of fatty oils mixed with all sorts of alkaline. This led to the conclusion that all cases of fatty oils mixed with all sorts of alkaline substances produce soap substance.

Take another illustration. All the members of a club are above 50 years old and none of them found any need to visit hospital since three years. In order to know the cause of this happy phenomenon, the method of agreement was applied. The common factor found among these people was that they do regular exercises and are leading a disciplined life.

Take yet another very common instance as an illustration of the method of agreement given by Irving M. Copi in slightly different way. Suppose in a party, after taking dinner, most of the guests got sick. In order to know the cause of sickness among the guests, the doctor employs the method of agreement (though he may not be aware of the fact that he is applying the method of agreement). He inquires about the food items the patients had taken. The doctor is trying to isolate a common factor, common food item which all the sick guests had consumed. The food items which the guests had taken were bread, curd, rice, mixed vegetables, salad and ice cream. After examining quite a few patients he found ice cream was the only food item taken by all sick patients. Consequently, the doctor reached to the conclusion that probably ice cream served to the guests was either a cause or part of cause of sickness among people.[1]

Mill explains the method as, "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon".[2]

The method can also be explained symbolically as:

Number of instances	*Circumstance (Cause)*	*Phenomenon (Effect)*
1.	ABCD	abcd
2.	AMNO	amno
3.	ALHP	alhp
4.	AXYZ	axyz
"	I	I
"	I	I
"	I	I
"	I	I
"	I	I

1. Cf. Irving M. Copi, and Carl Cohen, op. cit., pp. 487-88.
2. J.S. Mill, op. cit., bk. 3, ch. 8, sec. 18.

Therefore, probably A is cause or part of cause of a.

The symbolical expression of the method of agreement shows that it is essentially a method of elimination. It systematically eliminates the non-causal elements in the circumstances (cause) and consequently in phenomena (effect). One by one all factors like B, C, D, M, N, O, L, H, P, X, Y, Z were eliminated as the possible cause of the phenomenon a. Only A was isolated as a common circumstance and a was isolated as common phenomenon. Hence most probably A is the cause of a. One selects a common invariable and relevant factor occurring in these various instances and rejecting irrelevant or accidental factors so that the real and genuine relationship can be established.

The method of agreement is also called a method of observation. In this lies its merits as well as demerits. In the field where the experiments are very dangerous to perform, the method of observation is the only available method for the investigation of causal relationship. But sometimes the phenomenon occurs so rarely that one has to wait quite a long time to observe it again. It may be possible that the phenomenon may occur once in hundred years. In that case the method of agreement is not profitable method for discovering causal connection.

Moreover, Mill's formulation of the method demands that cause and effect events should be analysed into simpler components as shown in the symbolic expression. But how do we know that our analysis of the complex events into simpler components is correct? To an irrelevant factor we may wrongly consider as relevant and vice versa. How can we then be sure that in the analysis of component events, which factors are relevant for causation and which are irrelevant? The phenomena and circumstances do not come to us carrying tags of relevance or irrelevance. One has to take the help from other sources also in order to know the relevant factors in determining causal relationship. The method of agreement, it seems, by itself may not be sufficient to determine the causal relation among facts in some cases.

Let us consider here some of those cases where the method of agreement was applied but it did not establish the right causal relationship because irrelevant common factors were wrongly thought to be the cause of facts.

Instances	*Cause*	*Effect*
1.	Cat A crossed the path of x	It proved bad luck to x
2.	Cat B crossed the path of y	It proved bad luck to y
3.	Cat C crossed the path of z	It proved bad luck to z
4.	Cat D crossed the path of m	It proved bad luck to m

Therefore, crossing the path by a cat is the cause of bad luck to a man.

It is very clear the method of agreement was applied in the above example, but at the same time nobody will deny that the path crossed by a cat is not the cause of bad luck. It is sheer a chance that some accidentally common factor occurred again and again which is actually not the real cause. But how do we know which commonly occurring factors are genuine (and establish a causal relation) and which ones are accidental and cannot be causal indicator. It is only by some other source or sources, other than method of agreement, that we realize whether the common isolated factor is in fact the real cause or not. The difficulty in isolating the common relevant circumstances in some instances was brought out beautifully by Copi.[3] XYZ drank brandy and water and got drunk. XYZ drank gin and water and got drunk. Observing that water was present in each instance of the phenomenon under investigations, they concluded that water is the cause of drunkenness and solemnly resolved to avoid consumption of that liquid in future!

The major stumbling stone in the correct application of the method of agreement is that there is no way to differentiate the situation in which two facts are occurring together just by chance

3. Cf. Irving M. Copi, and Carl Cohen, op. cit., p. 511.

or whether they are coming together because of genuine relationship. It seems superstition and scientific knowledge both began from the same point, that is, by observing certain facts occurring together. In both the situations the togetherness of the phenomena is the basis of our belief in their occurrences. But whereas the former is not rooted in causal relation the latter cannot help being rooted in it. The method of agreement can merely suggest and can never prove the causal relationship.

Moreover, by the method of agreement one cannot differentiate between cause and co-existence. For instance, day and night follow each other invariably but none of them is cause of another. They both are rather the co-effects of something else. Similar objections can be made regarding heat and light, lightening and thunder.

Sometimes, the method of agreement takes us half way in the investigation of the causal relationships. For instance, in the case where people got sick after taking dinner, the doctor may isolate two food items as possible causes of sickness but out of them which one is the real cause of trouble, the method of agreement may not be able to locate. In that situation the doctor needs another method, the method of difference to determine the cause of sickness among people.

Method of Difference (Disagreement)

The method of difference can be applied to almost all those cases where the method of agreement is applied. But whereas the method of agreement requires large number of instances, the method of difference needs just two. The two instances should be such that they resemble each other in every respect except one. For instance, if a bell is rung in a jar filled with air, the sound of the bell is heard, but if the same bell is rung in a jar from which the air has been pumped out, no sound is heard (provided other circumstances remain the same). From this one concludes that the presence of air is an indispensable part of the cause of sound.

Take another illustration. Suppose a physician wishes to test the efficiency of a certain drug as a cure for cold. He will take an account of people suffering from cold and who do not take the drug and consequently continue to be ill for a fairly long time. The physician also takes into account of those patients who take the drug and get immediate relief. From these two different sets of patients the physician establishes the efficiency of the drug.[4]

In the case where people got sick after taking dinner, the doctor will take two such persons for inquiry who have taken the same food items except one. Two guests say P and Q have eaten B, C, D, L, M food items but whereas P has taken one more item A, Q has not taken that very item. From this the doctor concludes that probably the food item A is the cause of sickness among guests.

The canon of the method of difference stated by Mill is as "If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance save one in common, that one occurring only in the former, the circumstance in which alone the two instances differ, is the effect, or the cause, or a necessary part of the cause, of the phenomenon".[5]

Symbolically the method of difference can be expressed as follows:

Instances	*Cause*	*Effect*
1.	ABCERF	Sick
2.	ABCER	Not sick

Therefore, probably F is either a cause or part of cause of sickness.

The method of difference requires two such instances which resemble each other except in one respect. In the practical state of affairs, however, such a requirement is too difficult to meet.

4. Cf. Arthur Pap, op. cit., p. 153.
5. J.S. Mill, op. cit., bk. 3, ch, 8m sec. 2,

One cannot so easily find two events resembling one another in all aspects except one. This ideal condition is impossible to fulfil at times. But in experiments such rigorous requirement can be met because there the situation is under our control, and we can create the situation artificially according to our requirements.

If the method of difference is strictly applied in conformity with its requirements, then, Mill claims, it *proves* causal relation and thus yields conclusion with utmost certainty. Some logicians believe the method of difference gives "rigorous certainty". Out of the five of Mill's methods, this is the only method by which causal relations can be established and proved conclusively. But this is a "tall claim" regarding any of the experimental inquiry. In fact, neither inductive reasoning nor any of the experimental methods can ever prove the conclusion conclusively.

The method of difference, like all other methods, suffers from certain limitations. The method cannot be successfully applied in those cases where experiments are too dangerous to conduct.

Joint Method of Agreement and Difference

This is not a new method. Since the method of agreement and the method of difference are applied to the same cases, the investigator may sometimes use both of them to establish a causal relation.

The simultaneous application of twin methods increases our faith in the conclusion drawn from the evidences. The joint method thus is certainly superior than the method of agreement alone or the method of difference alone. Symbolically then joint method can be expressed as follows:

Instance	*Circumstance*	*Phenomenon*
1.	ABCD	abcd
2.	AMNO	amno
3.	AXYZ	axyz
"	I	I
"	I	I

Therefore, probably A is cause or part of cause of a,

and

Instance	*Circumstance*	*Phenomenon*
1.	ABCD	abcd
2.	BCD	bcd

Therefore, probably A is cause or part of cause of a.

Let us take once again the example of the guests who got sick after taking dinner in a party. The doctor applies dual method in order to be more certain about the poisonous food element. By the method of agreement he will make a list of patients and examine what each one had eaten:

Patients	*Food Items*	*Effect*
1.	ABCD	Sick
2.	AMCD	Sick
3.	AMCL	Sick
4.	ACBM	Sick
5.	ALDH	Sick

Therefore, probably A is either the cause or part of the cause of sickness.

The doctor makes another list of patients as follows:

Instance	*Food Items*	*Effect*
1.	BCD	Not Sick
2.	MCL	Not Sick
3.	CBM	Not Sick
4.	LDH	Not Sick

Therefore, probably A is the cause or part of the cause of sickness.

Combining the results of both the investigations the doctor finally reaches to the conclusion that food item A is most probably

the cause of sickness among guests. The joint method depends on the axiom that "whenever cause occurs effect occurs and whenever cause is absent effect is also absent". In the joint method the negative instances strengthen the conclusion drawn from the positive instances. Thus in this method there is emphasis both on necessary and sufficient conditions.

Method of Residues

Mill defined the method of residues as follows:

"Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents."[6] For instance, in order to know the weight of the cargo the usual practice is to weigh empty truck and then to weigh loaded truck of cargo. The difference between the weights is the actual weight of the cargo. Here the method of residues is applied, and except this method there is no other way to weigh the cargo.

Many times at a grocer's shop you would have noticed that the shopkeeper first weighs an empty container and then he weighs container again after filling it with ghee or oil. This is yet another illustration where the method of residues is applied.

The method of residues is very widely used in sciences. Many important scientific discoveries are made with the help of this method. The famous discovery of Madam Curie's new chemical radium and the tremendous radioactivity it produces is made with the help of the method of residues.

Similarly, the calculation of the path of the planet Uranus revealed the presence of new planet Neptune, in 1846. When the scientist Leverrier calculated the path of the planet Uranus then he found there is a difference of two minutes between the path calculated by him and the actual path taken by the planet which causes the difference in the calculation of the path. Later on it was further confirmed that there is actually another planet

6. Ibid., bk. 3, ch. 8, sec. 5.

Neptune which is the outermost of the all planets in the solar system.

Symbolically one can express the method of residues as follows:

Instance	*Circumstance*	*Phenomenon*
1.	ABCD	abcd
2.	B	b
3.	C	c
4.	D	d

Therefore, probably A is the cause of *a*.

The complex event ABCD is the cause of complex effect abcd; and previously it has been established that B is the cause of b, C is the cause of c, and D is the cause of d. From this acquired knowledge it is inferred that A is the cause of a, the residues circumstance is the cause of residues phenomenon. The leftover event is the cause of leftover effect.

Some thinkers regard the method of residues as the method of deduction because here less observation is done and more deduction and calculations are made. But this is not correct. For, it makes use of the knowledge of the causal relations which was inductively established earlier. Like all other experimental methods, the method of residues is also not a method of proof. The method takes us only a half-way and can be "a finger-post to the unexplained", but it can never demonstratively prove a causal connection.

Method of Concomitant Variation

The four experimental methods discussed so far are methods of elimination. In order to determine the causal relation the irrelevant circumstances are systematically eliminated. But there are circumstances which cannot be eliminated altogether. For instance, heat, gravity, atmosphere, friction, pressure, etc. are the natural necessities which cannot be eliminated altogether.

Though the complete elimination of these permanent causes is not possible, yet they vary in degrees, and through the method of concomitant variation one can establish the causal relationship between them.

Mill states the method of concomitant variation as: "Whatever phenomenon varies in any manner whenever some other phenomenon varies in some particular manner is either a cause or an effect of that phenomenon or is connected with it through some fact of causation."[7]

Symbolically the method can be expressed as follows:

Instance	*Cause*	*Effect*
1.	ABC	abc
2.	A+BC[8]	a+bc
3.	A-BC	a-bc

Therefore, probably A is the cause of *a*.

The two facts when vary together (either homogeneously or heterogeneously), they suggest the causal relationship between them.

The concomitant change in the temperature and mercury helped scientists to relate them causally. Similarly, the rise of tides in the sea is causally related with the increase in the size of the moon. As the size of the moon increases, the tides in the sea rise higher. In economics, the principle of demand and supply which determine the price of a commodity makes use of the method of concomitant variation. In order to know why cases of suicides,

7. Ibid., bk. 3, ch. 8, sec. 6.
8. Homogeneous change is that in which both the facts either increase together or decrease together, and heterogeneous change is that in which two facts vary inversely, that is, if one increases the other decreases.

   A+ means A increases.

   A- means A decreases.

rape, murders, robberies and other anti-social activities are increasing in society, one gathers the facts, events and also the situations which are increasing and hence are responsible for the rise in the undesirable behaviour. These examples make it clear that in the quantitative induction in method of concomitant variations is the only relevant method to determine causal relation.

However, this method like all other previous methods is not foolproof method. Sometimes two phenomena vary together but they are not causally related. They both may be the co-effects of some other cause. For example, the minute hand and the second hand in a watch vary simultaneously but the movement of hour hand is not the cause of the movement of minute hand or vice versa. Moreover, since the method of concomitant variation is the method of quantity, the method demands some scale to measure the increase or the decrease in the quantity. The scale may not be very accurate or sophisticated but certainly some scale is absolutely indispensable.

Assessment of the Methods

The study of the experimental methods may give an impression that the causal relationship among the natural phenomena can easily be established. This, however, is not always the case. The experimental methods are merely instruments, and how efficiently and successfully one makes use of these methods in discovering causal relationship, depends entirely on the skill and genius of the user.

Mill had made two claims regarding his experimental methods. *Firstly*, they are the methods of discovery. That is, these methods alone are sufficient and enough to establish causal relations among facts. The scientists' skill, wisdom and imagination, it seems, has no place in establishing causal relation. Something similar Francis Bacon has also stated. He said our method for discovering causal relation in "sciences is such as leaves but little to the accurateness and strength of wits, but places all wits and

understandings nearly on the level. For as in the drawing of a straight line or a perfect circle, much depends on the steadiness and practice of the hand, if it be done by aid of hand only, but it with the aid of rule or compass, little or nothing; so as it is exactly with any plan."[9]

Secondly, Mill claims the methods can *prove* the causal relations *demonstratively*. Inductive logic supplies, says Mill, "rules and models (such as the syllogism and its rules are for ratiocination) to which if inductive arguments conform these arguments are conclusive and not otherwise".[10] These rules and models are nothing but his experimental methods about which he claims that they have demonstrative function and has capacity to prove the causal relationship among facts just as a conclusion is proved in deductive logic.

But this is not correct. The experimental methods do not and cannot prove demonstratively the causal relationship. Since the methods make use of observations and experiments, the knowledge based on them can never be certain and demonstrative. Just as the inductive reasoning is problematic so also causal knowledge arrived through the experimental methods. Nothing in the inductive logic can claim to be demonstratively proved because inductive logic is not totally formal.

Thus both the claims which Mill made about his experimental methods are rather "big" claims, and cannot be accepted as valid. There is a "quest for certainty" in his theory of induction and because of this he made such claims. The experimental methods otherwise have an important function. They are extremely potential in suggesting a course of inquiry to a scientist, and they are important "milestones" and good "sign posts" on the roads of scientific discovery, though by themselves they are neither sufficient nor complete in establishing causal relations among facts.

9. Francis, Bacon, *Novum, Organum*, bk. 1, LXI.
10. J.S. Mill, op. cit., bk. 2, ch. 3, sec. 7.

Chapter 19

Hypothesis

In any scientific investigation, the scientist begins by assuming some possible explanation for the problem he is investigating. This possible explanation is called hypothesis. Using this as a starting point in his investigation, he proceeds to gather facts through observation and experience to find out whether the idea used in supposed explanation (hypothesis) is supported by them (experiences) or not. If our empirical evidences support the hypothesis, then the hypothesis is accepted as correct explanation, otherwise not. A hypothesis thus is merely a tentative or provisional solution to a problem. It is not the real solution of the problem (for which the hypothesis is constituted), till verified.

A hypothesis is of fundamental importance for a proper and systematic scientific investigation. It points the direction in which the scientist should make observations and conduct experiments. By framing hypothesis the research is focused on specific points and the entire energy (experiments and observations) is used to gather the evidences in favour of the hypothesis. Random investigations are of no help and only an organized investigation is fruitful. Hypothesis defines the scope of a scientific inquiry and helps in saving time and efforts of the investigators. The hypothesis serves as the starting point in the rigorous scientific research and thus a hypothesis is assumed as a guide to scientific inquiry. All scientific investigations start with the formulations of hypotheses and stops with their verification and proof. Historians, detectives, social scientists as well as natural scientists make extensive use of hypotheses.

It is not only in the branches of knowledge the hypotheses are framed, tested, verified and proved, but even in most ordinary and common practical life, we continually frame hypotheses in order to give an account for, or analyse or solve a practical problem. Suppose when I go back home and I find my son has not reached home from school which he normally does by that time, I start guessing and making tentative explanations as to what would have happened to him. I may suppose that he has missed the school bus, or his school bus is late for it is caught in traffic. These alternative explanations, suggestions or solutions are called hypotheses.

A hypothesis is simply a suggestion or a possible explanation of a particular phenomenon. It is merely a suggestive, tentative or a provisional solution to a problem. A hypothesis is not the real solution until tested and verified. All hypotheses are subject to revision, modification or even rejection.

Since a hypothesis is extremely important in every field of investigation, let us see how it is framed. The formation of a hypothesis is not a mechanical process. No rules or criteria can be laid down for making a "relevant" or "good" hypothesis. Actual framing of a hypothesis is the work of a genius. Here the sagacity, genius and originality of a scientist play important role, for only a genius or a trained mind can see something significant in the phenomenon which others do not notice. Hypotheses are suggested, writes De Morgan, not by rules, but by sagacity of which no description can be given previously because the very owners of it do not act under any prescribed laws. S.H. Mellone adds beautifully, "nature, herself, . . . does give broad hints as to the direction in which a fruitful hypothesis may be looked for, but only a prepared mind knows how to 'take the hint'4."[1]

Nature gives "broad hints" to inquisitive mind and they are "finger-posts" in nature pointing outlines of fruitful inquiry. Induction by simple enumeration and analogy are two main

1. S.H. Mellone, *Elements of Modern Logic*, p. 197.

sources which suggest hypotheses to a scientist. Two phenomena when repeatedly occur together, then it suggests that there is some connection between them, and consequently a scientist frames a hypothesis. In chapter 16, you have learnt that induction by simple enumeration cannot prove the causal connection between the two phenomena but it does suggest a hypothesis, a line of inquiry to a scientist. Similarly analogy does not conclusively prove a causal connection though it is a fruitful source for suggesting hypotheses. When it is found that two things resemble each other in certain important respects, we frame the hypothesis that they will possibly resemble each other in other respects too. The planets, earth and mars, resemble each other in possessing similar kind of atmosphere, land, etc. On the basis of these resemblances we are led to suppose that the planet mars might resemble the earth in being inhabited by living creatures. The analogy, thus, is a very important source of making hypotheses.

Science demands a higher degree of exactness and in this sense scientific research is a rigorous investigation. For this purpose there should be a well defined problem which needs a solution. One of the important characteristics of scientific thinking, and of all thinking having scientific spirit, is to grasp the problem and its nature before proceeding to solve it. Sometimes it comes to a scientist well defined and some other times the scientist himself sees a problem where others do not. Only attentive and intellectually curious mind sees the problem where others do not. A scientist then begins by providing possible solutions, conjectures, suppositions, hypotheses to solve the marked problem. All his energy is then focused on framing and verifying hypotheses.

Generally, a common man is inclined to treat any provisional supposition as an hypothesis. But a scientist does not consider seriously any supposed or provisional answer. A hypothesis is considered for serious discussion only if it satisfies certain conditions, though these conditions are, by no means, conclusive or exhaustive.

Conditions of Valid Hypothesis

The main characteristic of a "good", "valid" or "legitimate" hypothesis is that it should explain facts around us; it must be based on events actually occurring in nature. Valid hypothesis depends on facts in its origin and also for its verification.

Another important quality of a "good" hypothesis is that it should be verifiable with reference to observation or experience. The verifiability of a hypothesis can be done directly or indirectly. A hypothesis which lacks any verification (direct or indirect) is called barren hypothesis, and is not for the scientific inquiries.

Besides verifiability and having approved by actual experiences, a good hypothesis must satisfy other conditions too. A hypothesis should be purposeful, and directed to solve the problem for which it is framed. Random or irrelevant hypotheses cannot be considered for serious discussions in sciences or in ordinary common life. In other words, a hypothesis must be relevant to the problem it is supposed to solve. Relevance here means the hypothesis must either be a cause or part of a cause of a phenomenon for which it is formed. If, however, a hypothesis is neither of them, then it is irrelevant hypothesis and cannot be called "good" hypothesis.

Moreover, the hypothesis must be clearly and distinctly conceivable in itself. It should be stated in the clearest of terms. The vague, obscure and ambiguous hypothesis will only compound the problem of the scientist instead of solving it.

Generally a hypothesis which needs less calculation or mapping or observation is considered good. The scientist prefers the "mathematically simplest hypothesis" available. "For example, he will assume a linear equation rather than a quadratic equation if both fit the data".[2] Copernicus suspected that complexities of the epicyclic orbits of the heavenly bodies were due to the observer's own motion. By shifting to the sun as centre of the

2. Arthur Pap, op. cit., p. 226.

coordinate system, he managed to produce a far simpler system of celestial bodies. But simplicity should not be confused with familiarity.

One of the important characteristics of a good hypothesis is that it must not contradict other established truths or laws. This condition requires that we should take into account the achievements of the past. Certain things have been established rather definitely and any novel suggestion, which contradicts one of such well established laws, will be treated with suspicion. Knowledge of facts and laws previously ascertained gives steadiness to scientific knowledge, and any contradiction of these established laws will only raise the suspicion towards the new hypothesis.

But this condition at times can be violated and overruled. There are instances in the history of sciences when the established theories and cherished laws are abandoned in favour of new findings. For instance, Ptolemaic's[3] theory of universe (according to which the earth is the centre of the universe, and the sun, the moon and other planets revolve around it) was once established and accepted theory. It was later challenged and replaced by Copernican[4] theory according to which sun is the centre of solar system, and the earth, and other planets move around it.

A hypothesis which conforms to the various conditions laid down above is called "legitimate", "valid" or "good" or "working" hypothesis. A scientist goes ahead in his investigations with working hypothesis. A working hypothesis is accepted as true for time being and is used as a guide for further inquiry. But a working hypothesis is accepted as the real solution (to the problem for which it is constituted) only after verification and confirmation.

3. Claudius Ptolemy was an Egyptian astronomer who lived about CE127.
4. Nicolaus Copernicus was Polish astronomer who lived about CE 1473-1543.

Verification

A working hypothesis is to be tested and verified. Without verification a hypothesis can never be accepted as the real solution to the problem. This is the most crucial stage in the scientific investigation. There are two stages of testing the validity of a hypothesis.

1. Verification of a hypothesis
2. Proof of a hypothesis.

VERIFICATION OF A HYPOTHESIS

Verification of a hypothesis means testing of the truth of the hypothesis in the light of actual facts and in the context of empirical data. The process of verification involves a comparison between the conclusion deduced from the hypothesis and facts gathered through observation. The greater the agreement between the inferences derived from the hypothesis and facts gathered through observation or experience, the stronger will be the evidences in favour of confirming the hypothesis. Verification means to find out whether the conclusion derived from the hypothesis is supported by the actual experiences or not.

Verification of a hypothesis is of two types:

(i) Direct.

(ii) Indirect.

***Direct Verification*:** Direct verification is done by direct appeal to experiences and observations. Suppose one wants to test the hypothesis that the involvement of students in the college administration solves the discipline problem among the students. For this the behaviour of the students are studied to find out whether they respond to the responsibility given to them. If the observation shows that an increased participation in the college administration decreases the indiscipline among them, then the hypothesis is verified by direct observation.

***Indirect Verification*:** But in many cases direct verification of hypotheses is not possible. In those cases indirect verification, and indirect evidences are looked for to verify the hypotheses. In indirect verification the consequences are deduced from the hypothesis (the one which one wants to verify) and they are then compared with the actual facts. If the deduced consequences or evidences agree with the facts actually observed, then the hypothesis is verified, otherwise it stands rejected.

There are situations in science where a scientist verifies the hypothesis indirectly, that is, by deducing evidences. Arthur Pap says, "there is no other way of testing an empirical hypothesis — especially one of highly theoretical character, such as the hypothesis of universal gravitation, or the atomic hypothesis, or the gene theory of biology-than by deducing from it directly testable consequences."[5]

Indirect verification of a hypothesis is done by constructing a logical argument. If expressed in the strict logical form it becomes a hypothetical syllogism. Suppose one wants indirect verification of a hypothesis say H, then one will frame hypothetical syllogism as follows:

If the fact S is observed, then the hypothesis H is true.
S is observed.
Therefore, the hypothesis H is true.

Take a concrete example:

If the roads are wet, then it has rained recently.
The roads are wet.
Therefore, it has rained recently.

Take another example:

If the house is badly ransacked, then the aim of criminals was to steal.
The house was badly ransacked.
Therefore, the aim of criminals was to steal.

5. Arthur Pap, op. cit., p. 139.

In all the above examples the hypotheses are verified or confirmed on the basis of certain other facts which are observed as true.

The hypothetical syllogism can also be used to disconfirm or to disprove a rival hypothesis say H. The structure of the reasoning adopted for this purpose would be as follows:

If hypothesis H is true, then the fact S must be observed.
The fact S is not observed.
Therefore, the hypothesis H is not true.

Yet another example:

If it rained recently, then the roads are wet.
The roads are not wet.
Therefore, it did not rain recently.

Take another illustration:

If the aim of criminals is to rob, then the valuables must be missing.
But no valuables were missing.
Therefore, the aim of criminals was not to rob.

It is evident that by disproving the rival hypothesis, no hypothesis is proved. It merely eliminates rival hypotheses to increase the probability of some other hypothesis. For example, in order to know the cause of the gruesome crime in which all the members of the family were killed, the investigating team makes a number of hypotheses say H_1, H_2, H_3, etc. By framing hypothetical syllogisms the rivalry hypotheses are eliminated one by one. If the investigating team wants to know whether the hypothesis H_1, that is, the robbery was the motive of the crime, then by structuring hypothetical syllogism, as we have done above, H_1 can to be ruled out. The investigating team then takes second hypothesis H_2 for verification. The hypothesis H_2 states that probably the revenge was the cause of crime. The hypothetical argument is framed as follows:

If revenge was the cause of the crime, then the victims were known to be unpopular or unfriendly people.
But the victims were not known to be unpopular or unfriendly people.
Therefore, the revenge was not the possible motive of crime.

In this manner the hypothesis H_2 is also disproved and eliminated. The list of hypotheses is short-listed in this way and then the attention is paid to the remaining possible causes of the crime.

Scientists, detectives and historians use direct as well as indirect verification to justify and verify their hypotheses. The "utilitarian justification of a method of accepting and rejecting hypotheses"[6] is usually employed by them. The "utilitarian justification" of hypothesis means the "satisfaction produced" by accepting a particular hypothesis. If we get good results and it works pragmatically well in our experiences and explain the facts perfectly well, then the hypothesis is accepted as confirmed and verified. But no hypothesis is totally true or completely confirmed. There can only be a "high degree of confirmation".

PROOF OF A HYPOTHESIS

Verification (direct or indirect) is merely one step in the direction of proving a hypothesis. The hypothesis is called proved conclusively only when it stands as the only hypothesis, providing an adequate explanation for the facts or is the only solution for the problem. In verifying hypotheses, the scientist takes each alternative hypothesis one by one, and examines it in the light of available evidences. At this stage, no comparison is made among the rivalry hypotheses and as a result more than one hypothesis can be verified at a time. But while proving a hypothesis a scientist puts all the verified hypotheses together against each other to assess whether any one of them is supported by such conclusive evidences which are not available for the other

6. Ibid., p. 234.

hypotheses. Thus while verification is the process of evaluating each hypothesis separately on its own merits, proof is the process of comparison among the verified hypotheses. Proof of a hypothesis means a "process of elimination of competing hypotheses". After eliminating rivalry hypotheses, the best and the most acceptable hypothesis is chosen as the proved hypothesis. Hypotheses are first verified and then among verified hypotheses the best one is selected, and that chosen one is called proved hypothesis. When more than one hypotheses are verified then there is competition among them. In order to select one out of them, further investigations are done and then the best among them is chosen. This best chosen one is called *Proved hypothesis*.

A proved hypothesis must adequately explain all the facts for which it has been made and it must be the only hypothesis to do so. The hypothesis, say A, is called proved when it is the only explanation or cause of the fact say B. The fact B does not follow from any other source except A. "We must know not merely that A is an antecedent of B, but that A is the only possible antecedent of B . . ."[7]

Moreover, a proved hypothesis must explain not only those facts for which it is framed but other related facts also. For example, the law of gravitation explains not only the falling of bodies on the earth but provides the explanation for the movements of the planets and their behaviour. In short, a proved hypothesis must be verified, and it must be adequate to explain the phenomenon under investigation, and also it should be the only hypothesis to do so.

However, in science many hypotheses remain unproved even though they have been verified. This happens because the conclusive evidences may not be available to support the facts. For instance, there are different hypotheses regarding the origin of the world. Each one of them stands verified to some extent,

7. S.H. Mellone, op. cit., p. 195.

but none has yet been proved conclusively. The demand for absolute and conclusive proof of a hypothesis is an ideal demand which cannot be always met. However, the very hypothesis that stands verified with the support of evidences is raised to the status of theory. At the same time a well approved and generally accepted theory becomes a law.

Crucial Instances

As you have seen a hypothesis stands finally approved only when it eliminates the rivalry hypotheses and provides conclusively the most logical and complete explanation of the phenomenon under investigation. If a certain observation yields an instance which finally confirms one hypothesis and provides conclusive evidence against the others, such an instance is called crucial instance. The scientist stands on a cross road, wondering as to which hypothesis provides the real explanation for the phenomenon under investigation. Since an instance or an experiment helps in choosing his way, such an instance or experiment is called crucial.

Part IV

Chapter 20

Informal Fallacies

FALLACY is an error in reasoning, it is a mistake in judgement. An argument is governed by certain rules, and violation of any one of them makes the argument invalid and fallacious. Incorrect argument deceptively appears to be correct but on the examination is found incorrect. As you know the aim of a logician is to provide methods to differentiate correct reasoning from incorrect one, he thus investigates all the sources from where the incorrectness can creep into the reasoning. A logician picks up all the holes from where the errors in the reasoning enter and then suggests how to tackle them.

The list of errors to which a human mind is prone to, is long. In logic they are divided into two categories:

1. Formal Fallacies
2. Informal Fallacies

Formal Fallacies

A formal fallacy is committed when the argument has faulty structure or form. Formal reasoning such as syllogism, if it violates any of the rules of the valid categorical syllogism, then it commits the formal fallacy. Fallacy of undistributed Middle, Illicit Major, Illicit Minor, Existential Fallacy, etc. are a few instances of formal fallacies.

Informal Fallacies

But there are fallacies other than violations of the rules of formal logic. They are called informal fallacies and are committed by common man in his daily life. Though the logicians disagree about

the number and kinds of the informal fallacies, yet these fallacies are divided into two categories:

(i) Verbal Fallacies (Fallacies of Ambiguity)

(ii) Non-Verbal Fallacies of Matter (Fallacies of Relevance)

VERBAL FALLACIES (FALLACIES OF AMBIGUITY)

Fallacy of ambiguity arises due to the misuse of, or incorrect use of language. Some logicians call them the fallacies in speech or fallacies in diction. Since an argument is expressed through language, there can be many faults in expressing it. Hence, there are various verbal fallacies and some of the main fallacies in this category are:

(1) Fallacy of Equivocation

(2) Fallacy of Amphiboly

(3) Fallacy of Accent

(4) Fallacy of Composition

(5) Fallacy of Division

(1) *Fallacy of Equivocation*

Fallacy of Equivocation is similar to the fallacy of four terms in a syllogism. This fallacy is committed whenever we allow the meaning of a term to shift between the premisses or between the premisses and conclusion of an argument. The extra term is often created by using one of the three terms (major, minor or middle) in two different senses.

The simplest way in which the fallacy of ambiguity can be committed is by giving different meanings to the same term in an argument. This fallacy can best be shown by one of the many ridiculous examples that can be devised. For instance, one might agree that the enforced payment of income tax is a deprivation of freedom and since to be deprived of freedom is to become a slave, it implies that anyone compelled to pay income tax is a slave. Here the expression "deprivation of freedom" is equivocal. In the first premiss, "to be deprived of freedom" means a necessary

imposition in the interest of country, while in the second premiss it means a total deprivation of liberty.[1]

Look at another example:

All laws are made by governments.
v = at is a law of falling bodies.

Therefore, the government made v = at.

This is invalid reasoning, for the term "law" is unclear. At one place it refers to the physical law and at another place it is legislative law.[2] See yet another example:

If one has strong voice, the management should be handed over to him.

A wrestler has a strong voice.
So wrestler should be asked to take over the management.

Unintentional equivocation almost inevitably creeps into arguments on religion, philosophy, ethics and politics. Words like state, government, people, right, wrong, strong, good, bad, happiness and pleasure are understood in the context in which they are used and if the context changes so do their meanings.

Sometimes fallacy of equivocation is committed when we consider words derived from same root have the same meaning, or when a resemblance between ideas is established from the mere resemblance of words. For example:

The visible is what can be seen.
The audible is what can be heard.
Therefore, desirable is what can be desired.

It is true that "visible" means "what can be seen" and "audible" means "what can be heard", but "desirable" though it has the suffix "able" does not "mean what can be desired" but means "what ought to be desired".

1. Cf. E.W. Schipper and E. Schuh, *A first course in Modern Logic*, pp. 49-50.
2. Cf. *The World Book Encyclopedia*, vol. 12, p. 363.

Take another example:

Projector should not be trusted.
The engineer has formed a project
Therefore, the engineers should not be trusted.

This argument has fallacy of equivocation because the word "project" means "plan", whereas "projector" means a promoter of a questionable scheme.

Look to this example:

Idle men are inefficient.
Idle men are incapable.

Therefore, idle men are invaluable.

One wrongly assumes prefix "in" as "not" in all cases. Similar fallacy is committed when suffix "logy" (which means science) is attached to psychology, geology, biology, and on that basis one wrongly assumes astrology also a science of astros. But in astrology, "logy" is not a suffix at all. It is "astronomy", and not "astrology," which is science of "astros".

(2) *Fallacy of Amphiboly*

The fallacy of equivocation designates the ambiguity of a word or brief expression, whereas fallacy of amphiboly refers to the ambiguity of statement. The fallacy of amphiboly is due to grammatical structure of a sentence, and not due to multiple meanings of terms. In a famous amphiboly king Croesus was advised that if he attacks the Persians, he would destroy a "great empire". Whether the "great empire" was his own or that of Persia, the Oracle did not disclose. Croesus chose war with Persians and lost and thus destroyed his own "great empire"

Let us take another example. If we are told that a number is equal to three times five plus four, the statement is amphibolous, since the answer might be either

$[(3 \times 5) + 4 = 19]$ or $[3 \times (5 + 4) = 27]$[3].

3. Cf. E.W. Schipper and E. Schuh, op. cit., p. 53.

Amphibolous statements have always proved exceedingly useful to those persons who attempt to predict future events. Predictions are worded in such a way that they cannot be falsified. Indeed anyone wishing to gain a reputation as a seer, soothsayer, astrologist, palmist or something similar could not do better than to become skilful at devising amphibolous sentences. A sentence may be given more than one meaning when it is wrongly constructed. For this reason a hearer may take that meaning which suits him but which may not necessarily be true. It is obvious that all the different meanings given to the same statement in the same context cannot be true. For example, the famous amphibolous statement, "Alexander Darius shall conquer", may mean either "Alexander shall conquer Darius" or "Darius shall conquer Alexander". Clearly both of the meanings cannot be true at the same time but which one of them is true, the Oracle didn't disclose.

(3) *Fallacy of Accent*

Like the fallacy of amphiboly the fallacy of accent is not an inferential fallacy. The fallacy of accent is committed when the meaning of a statement is changed by wrongfully stressing words or other elements of the statements. The accent in question may be oral, or it may make use of italics, underlines, or other devices for accenting language. Often much of what we mean depends on the accent of words in an argument. In an attempt to derive home a point, we emphasize some words or phrases; but when a word which should not have been emphasized, is emphasized, then the meaning of the argument changes. Tone, accent and stress and italics can completely change the meaning of words and render an argument fallacious. For instance, consider the following example:

(a) *I* didn't plan to buy him a cricket bat.
(b) I didn't *plan* to buy him a cricket bat.
(c) I didn't plan to buy *him* a cricket bat.

Here the fallacy of accent is committed. The (a) proposition means that the speaker personally had no intention of buying a bat though someone else may buy bat for him; (b) proposition means the cricket bat was bought accidentally, and this was not the planned action of the speaker; whereas (c) statement means the speaker could probably buy a cricket bat to someone else but not to him.

When accent is employed either orally or in writing to clarify meaning, then it is being used in a correct way. But unfortunately sometimes it is used for insidious purposes. Propagandists deliberately distort statements to mislead audiences. For example tabloid newspapers often use bold and large print to attract the readers" attention. For instance, in a newspaper this type of advertisement can be seen very often – TAKE ABSOLUTELY FREE But somewhere it is written in a insignificant way, the contest entry form free. One should read fine prints also.

(4) *Fallacy of Composition*

Fallacy of composition is committed when an inference is made from the properties of the part of a whole considered individually (and separately) to the properties of the whole considered organically or collectively. What is true of each of the part may not be necessarily true for the whole. We commit the fallacy of composition whenever we argue from a premiss containing a term taken distributively to a conclusion in which that term is taken collectively.

Let us take an example of a cricket team. Whereas each player in the team may be an excellent player in his own individual right but that does not mean at the same time that the team as a whole is excellent. For a team in order to be good must possess qualities which render the team to have a smooth functioning, organically united and have a virtue of team spirit which is not applicable to good individual players.

Take another example where John Stuart Mill commits this fallacy when he argues that the general happiness is the greatest

good. "No reason can be given why the general happiness is desirable, except that each person . . . desires his own happiness . . . each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons".[4] In other words, my happiness is my good, your happiness is your good, his happiness is his good, x's happiness is x's good. Though the argument is persuasive yet it commits the fallacy of composition. The truth of the second clause of the last sentence does not follow from the truth of the first. The good of each person may be his own happiness when taken distributively, but it is not, therefore, the case that the general happiness stands as good to all persons taken collectively.

(5) *Fallacy of Division*

This fallacy is simply the opposite of the fallacy of composition. It consists in predicating the qualities to the parts which can be safely predicated only to the whole. In such an argument we wrongly take separately what we ought to take jointly. To use the same example as before, one cannot reason that since a given cricket team is a good team, each of its players must, therefore, be good. Numerous illustrations of both the fallacies of composition and division can be found in the arguments concerning the interests of the nation and citizens. Many persons reason wrongly that what is in the interest of individuals must also be in the interest of nation, thus committing the fallacy of composition. Others commit converse fallacy by arguing that what is best for the nation must necessarily be advantageous for each person.

NON-VERBAL FALLACIES OF MATTER (FALLACIES OF RELEVANCE)

Non-verbal fallacies are committed when the premisses of the argument are found irrelevant to the conclusion for one reason or the other. The premisses appear to be relevant to the

4. J.S. Mill, *Utilitarianism, Liberty, Representative Government*, Chap. IV, pp. 32-33.

conclusion but on the close examination are found either inadequate or incomplete. The premisses may appear to be psychologically relevant, but for a sound argument the premisses must be logically, and not psychologically relevant.[5]

Non-verbal fallacies or material fallacies as they are sometimes called, are committed "outside the language" in contrast to the fallacies of ambiguity which lie "in the language". It is the matter and not the form of an argument which is responsible for the fallacy. However, the material fallacies should not be confused with errors of fact or factual errors; the latter would be false statements and not fallacious arguments.

The premisses can be irrelevant to the conclusion in various ways and hence there are many non-verbal fallacies. The main fallacies in this group are:

(1) The Argument from Ignorance (Argument *Ad Ignorantiam*)
(2) The Appeal to Inappropriate Authority (Argument *Ad Verecundiam*)
(3) The Argument Against a Person (Argument *Ad Hominem*)
(4) Fallacy of Accident
(5) Fallacy of Converse Accident
(6) False Cause (*non causa pro causa*)
(7) The Appeal to Emotion (Argument *Ad Populum*)
(8) The Appeal to Pity (Argument *Ad Misericordiam*)
(9) The Appeal to Force (Argument *Ad Baculum*)
(10) Irrelevant Conclusion (*Ignoratio Elenchi*)
(11) Begging The Question: *Petitio Principii*
(12) Complex Question

5. Irving M. Copi, and Carl Cohen, op. cit., p. 116.

In the first ten fallacies, the premisses do not imply the conclusion because they are not relevant to the conclusion. In the last two fallacies the error is there because premisses are either trivial or inadequate to the conclusion. Let us discuss each fallacy in detail.

(1) *The Argument from Ignorance*

Sometimes the lack of evidences regarding an event or a fact have been used as evidences in favour of the argument. For example, "Ghosts do not exist" is true because so far no one has definitely proved its existence. Since no one has proved so far conclusively that "Ghosts exist", therefore, it can be concluded that "Ghosts do not exist". The argument about the non-existence of ghosts is based upon an appeal to our ignorance instead of knowledge. But the fact that we could not definitely prove so far the existence of ghosts, that does not mean conversely the non-existence of ghosts. It seems in this fallacy the strength of our argument (the basis of the conclusion) is our ignorance or lack of proper evidence either in favour or against the fact instead of knowledge.

The fallacy usually falls into two categories:

(i) Arguing from the absence of proof to the presence of disproof. The above example falls in this category.

(ii) Arguing from the absence of disproof to the presence of proof.[6] The following example falls in this category. An atheist may argue that since no one has disproved a supernatural being, hence the supernatural being exists.

In daily life we often come across this kind of fallacious reasoning. The customers often ask the shopkeeper about the quality of a particular thing or item, say for instance cloth. The shopkeeper with equal eagerness replies that so far no one has complained against it, and this is a good enough proof for accepting the quality of cloth. Similarly a rather too conscious

6. Cf. E.W. Schipper, and E. Schuh, op. cit., p. 32.

customer asks the manufacturer about the durability of the glass he is buying. The manufacturer's standard answer is that I have been making and selling glass for nearly ten years and since then I have not heard any complaint against my product and this is good evidence to believe in the quality of the product.

(2) *The Appeal to Inappropriate or Misplaced Authority*

One very common and crafty fallacy occurs if a person who is an expert authority in one field is taken as an expert authority in some other, comparatively, unrelated field. This fallacy is difficult to avoid at times and difficult even to expose, because some men achieve so great a status in their own field that their prestige overflows into other fields. For instance, Bertrand Russell, the greatest philosopher of the twentieth century, is an unquestionable authority in the philosophical matters. But if he says that a particular shoe is good or is worth buying, then his statement is not authoritative. For, he is not an expert in the field of shoes. Only those who deal in these areas and have gained knowledge and experience are expert, and their words and statements should be treated as reliable.

But often the advertisers in the haste to promote their products, commit this error. For instance, since Sachin Tendulkar is saying a particular drink is good, so one must take his words and accept that particular brand of drink as superb. But this is wrong. Sachin is no authority in the field of cold drinks. He is authority in cricket. If he had recommended a particular brand of cricket ball or a cricket bat, then his words would have been reliable and genuinely authoritative. But in the field of cold drinks he is no better than any other man.

Sometimes the fallacy of inappropriate authority is committed in spite of our best efforts. For instance, a scientist who is expert in manufacturing nuclear weapons is no authority in the international economical matters. But his words are wrongly taken as an authority in these matters. Similarly a religious leader may not be an expert in financial matters.

(3) *Argument Ad Hominem*

The fallacy of argument *ad hominem* is committed when the argument is examined not on the basis of the relevance of the premisses to the conclusion, but on the basis of who is saying it or in what circumstances one is making an argument. Personal attitudes, emotions, prejudices or some kind of subjectivity is the main source of this fallacy. The argument is judged on the basis of the personal assessment rather than on the merit of the premisses. The conclusion of an argument is denied not because premisses are inadequate or non-meritorious but because it is stated by a person who either carries a bad reputation or belongs to opponent camp or his circumstances are such that it compels an arguer to hold a particular view. But the character, the nationality, the religion which an arguer holds, are not relevant at all to the truth or falsity of the conclusion because the soundness of an argument is neutral to all the subjective conditions.

The conclusions in the argument *ad hominem* are often *denied* or *rejected* (though denied or rejected on wrong footing) whereas the conclusion in the fallacy of inappropriate authority are *accepted* (though again on wrong footing).

In the simplest language, the fallacy of argument *ad hominem* is committed in the following manner. Person A makes an argument. Person B evaluates that argument. Person B shows that the argument made by A is wrong on the ground that either (1) person A carries a bad reputation and hence his argument cannot be sound, or (2) person A belongs to a particular group, or (3) person A's circumstances are questionable.

It may be possible that argument made by person A is unsound but the reasons given by Person B is certainly fallacious. Person B attacks the argument made by A on the personal grounds instead of on the strength of the premisses in respect to the conclusion.

Politicians argue *ad hominem* when they attack the opponents' policies by attacking their motives and characters instead of examining policies on their merits. Instead of evaluating the policies upheld by the opposite political parties on the rational ground, the rejection by the ruling party is purely subjective and biased.

There are various forms of this fallacy. In the abusive form, the conclusion of the argument is denied or rejected not because the premisses are inadequate to support the conclusion but by attacking the person who is making an argument. For example, I am surprised to hear that you advocate an increase in liquor tariff. Were you not yourself in favour of the anti-drink campaign only recently?

Second form of this fallacy is circumstantial. Sometimes the conclusion of an argument is denied on the basis of profession, nationality, religion or the political affiliation of an arguer. For example, since he is Indian, so he must be religious man. Here the fallacy of argument *ad hominem* occurred because a man's special circumstances are taken as a reason for accepting the truth of some statement.

The fallacy of argument *ad hominem* may take the form of *genetic fallacy*. Genetic fallacy involves rejection of an idea or statement in its origin. If a man is shown to be a "congenital liar", then this could serve to discredit whatever he says in future.

Another form of the fallacy of argument *ad hominem* is called "poisoning the well". This can be regarded as a special type of genetic fallacy. To poison the well is to discredit the argument in advance without objectively analysing the evidences. For example, I would not believe Ram Devi's statement regarding the presence of the criminal in the courtyard because she is too old to see accurately, or I wouldn't believe Ram Prasad's statement that he quickly ran to window to see who is jumping out of the house, because he (Ram Prasad) uses wheelchair to move. But, in spite of these weaknesses, it may be possible that Ram Prasad's statement is true.

There is yet another form of fallacy of argument *ad hominem*. It is called "Tu quoque" which means "look who is talking". The minister is advising everyone to pay taxes honestly whereas he himself is a prominent defaulter in paying taxes.

Whatever may be the form of argument *ad hominem*, it is certain that this fallacious type of reasoning is employed by the person who is solely concerned with his opponent rather than with the merit of his argument. The soundness of an argument is to be seen irrespective of personal prejudices and self-interest.

(4) *Fallacy of Accident*

The fallacy of accident consists in applying general rule to a particular case in which "accidental" circumstances make the rule inapplicable to it. The fallacy is generally committed when a general rule is applied mechanically to all cases without any exception. We argue fallaciously when we apply a general rule without any regard to the special circumstance which may change its applicability in that case. Human needs, human values, human circumstances demand the modification of general rules in certain special cases. For example:

To charge interest on the money loaned is quite legitimate. Therefore to take interest loaned to a friend in distress is quite legitimate.

There is fallacy of accident in this argument. Usually it is not wrong to take interest on the money loaned but it is not fair to take interest on the money given to a needy friend.

In the realm of morals, the fallacy of accident occurs if one is not careful in applying the general moral dictum. For instance, it is true lying is sin but in order to save one's life, it is not wrong to lie either. It is wrong to steal but in order to save a starving man, stealing a loaf from someone who does not need it, is permitted. The theologicians, moralists and lawyers have to consider the distinction between general rule and a special circumstance with the greater care. Logician's job is to warn the arguer that the fallacy can creep in reasoning when we argue

from an unqualified statement to a qualified one. Look at the following example:

All killers of men are murderers.
Soldiers are killers of men in war.
Therefore, the soldiers are murderers.

This is a fallacious argument, for murder is not simply killing, but taking human life wrongfully. All killings, however, are not murders.

(5) *Converse Fallacy of Accident*

This fallacy occurs when we pass from a statement which is true in certain special condition to a statement which is supposed to be true under all conditions. For example:

To give charity to young healthy beggar is wrong.

Therefore, Charity is bad.

Giving charity to a able-bodied healthy beggar is certainly wrong but that does not mean all kinds of charity are wrong.

(6) *False Cause*

This fallacy occurs when we wrongly assume a non causal event to be either cause or part of cause of an effect. The fallacy of False Cause occurs when a causal relationship is assumed to exist when actually there is none. Superstitions, for example, suffer from the fallacy of false cause. For example: A black cat crosses the path of a traveller and shortly afterwards the traveller broke his head. The path crossed by the black cat was taken as the cause of the traveller's broken head. But we all know that the crossing of path by a black cat is not the cause of man's breaking head.

Sometimes this fallacy is also referred as the fallacy of *non causa pro causa*. If to a non-causal event wrongly we assume to be the cause, then there is fallacy of *non causa pro causa*.

Post hoc ergo propter hoc is another form of the fallacy of false cause. The fallacy means "after the thing, therefore, because of the thing". It is true, the cause is always antecedent to the effect in the time sequence, but to say every or any antecedent event is the cause of the following event is wrong. Simply because one event follows another does not mean that they are related by causal relationship. For instance, when the house got burnt, sun was shining very bright, therefore, to argue that shining sun is the cause of the house burning is fallacious and suffers from the fallacy of *post hoc ergo propter hoc*.

Let us take another related example. The house got burnt because Mr X was smoking in the room. Therefore, Mr X's smoking in the room was the cause of the house burning. Now it may be possible Mr X threw the cigarette carelessly in the house and it might have spread the fire there. But before we think like that, we must carefully examine the antecedent events. *Not every antecedent event is necessarily the cause of the following event.* However, in order to ascertain the cause of certain phenomenon, it may be possible that in spite of the scientists', best efforts, genuine error of false cause is committed in some cases.

(7) *The Appeal to Emotion*

In the argument *ad populum*, the conclusion of the argument is accepted on the basis of sentiments and emotions. Here reasoning regarding important issues is made by appealing either to masses or to the emotions. In such cases the premisses are so chosen as to be the instruments with which to manipulate the beliefs of the masses.

The fallacy of argument *ad populum* is committed by anyone who addresses a mass audience and tries to win over them by arousing emotions. A religious leader arouses the passions of his audiences in order to win over them, and similarly a politician defends himself against the accusation of taking bribes from contractors by arguing to the war records of his family, his goodness to the needy, and his patriotic records.

Sometimes an issue is emotionalized whereas some other times "emotionally toned words" are used in order to win over them.

Look at the following emotionalized argument:

India should protect its sovereignty at all costs. With so much of sacrifices we got our freedom. Our forefathers had suffered inhuman treatment to achieve independence. In this respect every Indian should support all the military efforts to preserve the freedom and integrity of the motherland. Therefore, it is highly appreciable that in order to protect our freedom, we go in for possessing nuclear weapons.

Here the appeal is made to heart, and not to head. There can be other valid reasons also for concluding that India needs nuclear weapons other than flaming the people's emotions by arousing patriotic feelings. The soundness of reasoning should be examined not on the basis of emotive or expressive language of premisses, but on the relevance of the premisses to the conclusion.

The advertising agencies though do not use "emotionally toned words" yet they sometimes commit this error. The products are displayed in such a manner as to arouse the approval of the masses. Bakery products like bread and biscuits are shown consumed by smiling, chubby boys and girls. The young charming and smart girls are shown using a particular brand of shoes or jeans, etc. Uses of certain brand of cars are related with the symbol of richness, whereas use of certain drink is shown as the sign of adventurism. Sleek furniture is shown used by sophisticated officers. The sole idea behind all this is to influence the attitude of public, and so strong is the influence that in spite of all the efforts to resist this, one is carried by them.

(8) *The Appeal to Pity*

The argument having the fallacy of appeal to pity is a special case of the argument based on the appeal to emotions. This is a persuasive type of argument in which the relevance of the premisses to the conclusion is not considered, but instead the argument is evaluated totally on the basis of pity and sympathy. Usually such type of emotional blackmailing is done when all

the valid and legitimate grounds for holding the conclusion are exhausted, or when the defence attorney is unable to offer any good reasons why his client is not guilty.

Its amazing the extent to which the argument *ad misericordiam* is used by all sections of people. While collecting charity funds, or funds for any noble cause, the argument *ad misericordiam* is often used as forceful means. Also in the court in a bid to win the favour of jury, the case is presented in such a manner as to arouse sympathy and pity in their minds. The situation is so presented as to appeal to heart rather than to head.

While pleading the case of reservations in the government jobs for the under privileged, the argument to pity is used for the approval of the jury. On "mercy appeal" the President of India can reduce the punishment of the accused as per Constitution of India.

Often it is seen that in obtaining compensation for a poor family of an accident victim every effort is made by the attorney to get the maximum amount by the appealing to mercy.

(9) *The Appeal to Force*

The threat of punishment or force of some sort is sometimes used for persuading the opponents to accept certain views. The appeal to force does not necessarily have to involve a physical threat like, "If you do not act like this, then I will hurt you". Sometimes there are "subtle" threats which force others to accept certain views or opinions. Anyone who tries to persuade the opponent to accept his dictate by threatening, is guilty of using the argument to force. It is very old saying "When you have no case, well get angry and threaten". In other words, the best policy to defend yourself is to become offensive.

Though it is barbaric rule to use force for getting one's views accepted, yet in civilized society of ours, "subtle" and not so open threats are used as weapons of persuasion. Right from the domestic front to the international forum, the "subtle" threat is

used as a powerful instrument. For example, the father tells his son, "Sunny, next time before giving you pocket money, I will see your report card". Many powerful nations are using "arms twisting" policies like reducing financial aid, cutting the technical assistances, etc. if the opponent countries do not sign a particular treaty. The union workers threat the establishment that if their demands are not met they will go on strike.

In spite of the fact that the threats are used implicitly on a large scale by all sections of society, yet logically to accept a conclusion merely on the basis of threat (and not on the basis of reason), is not sound.

(10) *Irrelevant Conclusion*

There are many arguments where the premisses are irrelevant to the conclusion but they do not fall in any of the fallacies considered so far. All those arguments are thus considered under the fallacy of irrelevant conclusion. The fallacy is committed when the premisses do not imply the conclusion which they are supposed to, and instead, they imply something else. The following example explains the fallacy better:

The object of war is durable peace; therefore, soldiers are the best peacemakers. Even if it is assumed that the object of war is durable peace, still it does not imply that soldiers are best peacemakers.[7]

(11) *Fallacy of Petitio Principii*

The fallacy of *petitio principii* is committed when the conclusion is used as an evidence. It is used as a premiss. If the conclusion is already assumed in the premisses, then the argument certainly becomes fallacious. The premisses are not irrelevant to the conclusion but they seem to be trivial or superfluous. If we assume what requires proof, then we are just beggars, begging what we ought to earn by proof.[8]

7. C. Read, *Logic – Deductive and Inductive*, p. 324.
8. Cf. A.A. Luce, *Teach Yourself Logic*, p. 168.

However, the fallacy is not committed deliberately but the eagerness to prove the conclusion is so strong that one unconsciously assumes (as an evidence) what one wants to prove.

Richard Whately's fallacy of *petitio principii* is quoted by I.M. Copi as follows:

> To allow every man unbounded freedom of speech must always be, on the whole, advantageous to the state; for it is highly conducive to the interests of the community that each individual should enjoy a liberty, perfectly unlimited of expressing his sentiments.[9]

A celebrated example of the fallacy which occurs in the philosophical writings is the argument that every thing in the world has a cause, since if it did not, we should have effects without causes.[10] Here the fallacy is more subtle than in the example given above, for it assumes rather than restate conclusion.

J.S. Mill was of the opinion that Aristotelian categorical syllogisms commit the fallacy of *petitio principii*. For example:

All men are mortal.
Ram is a man.
Therefore, Ram is mortal.

While establishing the truth of major premiss, "All men are mortal", the conclusion, "Ram is mortal", is already taken into consideration. The conclusion, since being used as an evidence in the syllogism, commits the fallacy of *petitio principii*.

But the syllogism may not necessarily commit this fallacy; for while enumerating the general proposition like "All men are mortal", the conclusion "Ram is mortal" need not be taken into account. The universal propositions are established by incomplete enumeration.

9. Irving M. Copi, and Carl Cohen, op. cit., p. 127.
10. R. Clark, and P. Welsh, *Introduction to Logic*, p. 143.

The example of a non-syllogistic argument having the fallacy of *petitio principii* would be the case of the man who registered a woman at a hotel as his wife and who replies, when asked for proof, "Certainly she's my wife; I am her husband."[11]

(12) *Complex Question*

This fallacy is committed when a question is asked in such a manner that a particular belief or assumption has been accepted as the basis for answering the question. For example: Exactly how did you feel when you murdered your brother?[12] If the fellow never murdered his brother, he would hardly know how to deal. The question is complex because it assumes and takes for granted that he is a murderer.

Complex question is favourite device of lawyers who wish to confuse a witness or defendant. For example, a lawyer asks a youth, reply in Yes or No — Are you still a leftist? Here many questions are implicitly woven into one. For instance: (1) Were you ever a communist? If the answer to the question is yes, then (2) have you left it now? The lawyers generally would ask such questions purposely to trap a defendant into making an implicit admission of his guilt.

When the question is complex, the best way to avoid the fallacy is to refute all the presuppositions hidden in the question one by one. To a question which appears to be simple but actually implies many questions, a straight answer will lead to a position which actually the replier does not hold. For example: "Why are our politicians so corrupt?" "How can we change our education system to make our studies more effective"? "Why are children so disobedient to elders", are instances of complex questions.

The informal fallacies discussed above, both verbal and non-verbal, are by no means exhaustive. There may be fallacious

11. Ibid., p. 143.
12. Ct. Cf. Kaminsky and Kaminsky, *Logic: A Philosophical Introduction*, p. 42.

reasoning which may not fall under any of the above discussed categories of fallacies. But it is absolutely necessary that one should make right and straight reasoning in order to avoid confusion. The saying:

Be bright, think right

has a point.

Select Bibliography

Alexander, Peter, *An Introduction to Logic*, London: George Allen and Unwin Ltd, 1971.

Barker, Stephen F., *The Elements of Logic*, New York: McGraw-Hill Book Company, 1965.

Basson, A.H. and D.J. 'O.Connor, *Introduction to Symbolic Logic*, London: University Tutorial Press, 1953.

Brennan, Joseph Gerard, *A Handbook of Logic*, New York: Harper and Brothers, Second Edition, 1961.

Carney, James D, *Introduction to Symbolic Logic*, New Jersey: Prentice-Hall, 1970.

Chomsky, Noam, *Knowledge of language (Its Nature, Origin, and Use)*.

Clark, Romane and Paul Welsh, *Introduction to Logic*, New York: D. Van Nostrand Company, 1962.

Cohen, Morris R. and Ernest Nagel, *An Introduction to Logic and Scientific Method*, New Delhi: Allied Publisher, 1972.

Copi, Irving M., *Introduction to Logic*, New York: Macmillan Publishing Co., Sixth Edition, 1982.

Copi, Irving M. and Carl Cohen, *Introduction to Logic*, New Delhi: Prentice Hall, Ninth Edition, 1995.

Crystal, David, *Cambridge Encyclopedia of Language*, UK: Cambridge University Press, Second Edition, 1997.

Dumitriu, Anton, *History to Logic*, New Delhi: Heritage Publishers, 1991.

Harman, Gilbert, *Thought*, Princeton, New Jersey: Princeton University Press, 1973.

Kammuisky, Jack and Alice Kammuisky, *Logic: A Philosophical Introduction*, United States of America: Addison–Wesley Publishing Company, 1974.

Kneale, W., and M. Kneale, *The Development of Logic*, New York: Oxford University Press: 1964.

Kreyche, Robert J., *Logic for Undergraduates*, New York: Holt, Rinehart and Winston, Third Edition, 1970.

Latta, Robert and Alexander Macbeath, *The Elements of Logic*, London: Macmillan 1956.

Luce, A.A., *Teach Yourself Logic*, London: The English Universities Press, 1968.

Mellone, S.H., *Elements of Modern Logic*, London: University Tutotial Press, Second Edition, 1970.

Mill, John Stuart, *A System of Logic* (1843), London: Longmans, Green and Co., 1925.

Pap, Arthur, *An Introduction to the Philosophy of Science*, New York: The Free Press, 1962.

Quine, W.V., *Methods of Logic*, London: Routledge and Kegan Paul, Third Edition, 1974.

Russell, Bertrand, "Logic as the Essence of Philosophy," *Essays in Logic from Aristotle to Russell*, (ed.) Ronald Jager, U.S.A.: Prentice Hall, 1963.

Schipper, Edith Watson and Edward Schuh, *A First Course in Modern Logic*, London: Routledge and Kegan Paul, 1960.

Stebbing, L. Susan, *A Modern Elementary Logic*, London: University Paperbacks, Methuen, 1961.

Stebbing, L.S., *A Modern Introduction to Logic*, New Delhi: Asia Publishing House, 1961.

Strawson, P.F., *Introduction to Logical Theory*, London: Methuen and Co., 1952.

The World Book Encyclopedia, USA, 1995.

Wittgenstein, Ludwig, *Philosophical Investigations*, trans. G.E.M. Anscombe. Oxford: Basil Blackwell; New York: Macmillan, 1958.

Wolf, A., *Textbook of Logic*, London: George Allen and Unwin, Second Edition, 1961.